In the previous chapter, we considered the semiconductor in equilibrium and determined electron and hole concentrations in the conduction and valence bands, respectively. A knowledge of the densities of these charged particles is important toward an understanding of the electrical properties of a semiconductor material. The net flow of the electrons and holes in a semiconductor will generate currents. The process by which these charged particles move is called transport. In this chapter we will consider the two basic transport mechanisms in a semiconductor crystal: drift—the movement of charge due to electric fields, and diffusion—the flow of charge due to density gradients. We should mention, in passing, that temperature gradients in a semiconductor can also lead to carrier movement. However, as the semiconductor device size becomes smaller, this effect can usually be ignored. The carrier transport phenomena are the foundation for finally determining the current-voltage characteristics of semiconductor devices. We will implicitly assume in this chapter that, though there will be a net flow of electrons and holes due to the transport processes, thermal equilibrium will not be substantially disturbed. Nonequilibrium processes will be considered in the next chapter.

The drift of electrons and holes caused by an electric field produces a drift current, which is a function of the concentrations of the mobile electrons and holes, and also a function of the net drift velocity of these charge carriers. The average drift velocity of a carrier is related to the electric field by a parameter called mobility. Electrons and holes do not move totally unimpeded through a semiconductor, but are involved in collisions with semiconductor atoms and with ionized impurity atoms. The mobility gives an indication of how well a carrier moves in a semiconductor.

Diffusion is the process whereby particles flow from a region of high concentration to a region of low concentration. If these particles are charged, as are electrons or holes, then diffusion of these particles leads to a diffusion current. The diffusion current density is related to the gradient of
the carrier concentration by a parameter called the diffusion coefficient. The diffusion coefficient gives an indication of how well a particle will diffuse through a semiconductor due to a density gradient.

Finally in this chapter, we will discuss the Hall effect. The Hall effect involves the forces on a moving charged particle due to electric and magnetic fields that are perpendicular to each other. The Hall effect is used extensively to experimentally measure the majority carrier concentration and carrier mobility. The Hall effect is also used in computer keyboards as a Hall-effect sensor.

5.1 CARRIER DRIFT

An electric field applied to a semiconductor will produce a force on electrons and holes so that they will experience a net acceleration and net movement, provided there are available energy states in the conduction and valence bands. This net movement of charge due to an electric field is called drift. The net drift of charge gives rise to a drift current.

5.1.1 Drift Current Density

If we have a positive volume charge density \( \rho \) moving at an average drift velocity \( v_d \), the drift current density is given by

\[
J_{df} = \rho v_d \tag{5-1}
\]

where \( J \) is in units of coul/cm²-sec or amps/cm². If the volume charge density is due to positively charged holes, then

\[
J_{phdf} = (e \rho) u_{dp} \tag{5-2}
\]

where \( J_{phdf} \) is the drift current density due to holes and \( u_{dp} \) is the average drift velocity of the holes.

The equation of motion of a positively charged hole in the presence of an electric field is

\[
F = m_p^* a = eE \tag{5-3}
\]

where \( e \) is the magnitude of the electronic charge, \( a \) is the acceleration, \( E \) is the electric field, and \( m_p^* \) is the effective mass of the hole. If the electric field is constant, then we expect the velocity to increase linearly with time. However, charged particles in a semiconductor are involved in collisions with ionized impurity atoms and with thermally vibrating lattice atoms. These collisions, or scattering events, alter the velocity characteristics of the particle.

As the hole accelerates in a crystal due to the electric field, the velocity increases. When the charged particle collides with an atom in the crystal, for example, the particle loses most, or all, of its energy. The particle will again begin to accelerate and gain energy until it is again involved in a scattering
process. This continues over and over again. Throughout this process, the particle will gain an average drift velocity which, for low electric fields, is directly proportional to the electric field. We may then write

\[ v_{dp} = \mu_p E \] (5-4)

where \( \mu_p \) is the proportionality factor and is called the hole mobility. The mobility is an important parameter of the semiconductor since it describes how well a particle will move due to an electric field. The unit of mobility is usually expressed in terms of \( \text{cm}^2/\text{V-sec} \).

By combining Equations (5-2) and (5-4), the drift current density due to holes may be written as

\[ J_{p\text{def}} = (ep)v_{dp} = e\mu_p E \] (5-5)

The drift current due to holes is in the same direction as the applied electric field.

The same discussion of drift applies to electrons. We may write

\[ J_{e\text{def}} = \rho v_{en} = (-en)v_{en} \] (5-6)

where \( J_{e\text{def}} \) is the drift current density due to electrons and \( v_{en} \) is the average drift velocity of electrons. The net charge density of electrons is negative.

The average drift velocity of an electron is also proportional to the electric field for small fields. However, since the electron is negatively charged, the net motion of the electron is opposite to the electric field direction. We can then write

\[ v_{en} = -\mu_e E \] (5-7)

where \( \mu_e \) is the electron mobility and is a positive quantity. Equation (5-6) may now be written as

\[ J_{e\text{def}} = (-en)(-\mu_e E) = e\mu_e nE \] (5-8)

The conventional drift current due to electrons is also in the same direction as the applied electric field even though the electron movement is in the opposite direction.

Electron and hole mobilities are functions of temperature and doping concentrations, as we will see in the next section. Table 5-1 shows some typical mobility values at \( T = 300^\circ \text{K} \) for low doping concentrations.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \mu_e \text{(cm}^2/\text{V-sec)} )</th>
<th>( \mu_p \text{(cm}^2/\text{V-sec)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>1350</td>
<td>480</td>
</tr>
<tr>
<td>Gallium arsenide</td>
<td>8500</td>
<td>400</td>
</tr>
<tr>
<td>Germanium</td>
<td>3900</td>
<td>1900</td>
</tr>
</tbody>
</table>
Since both electrons and holes contribute to the drift current, the total drift current density is the sum of the individual electron and hole drift current densities, so we may write

$$J_{df} = e(\mu_n n + \mu_p p)E$$

(5-9)

**Example 5-1**

**Objective:** To calculate the drift current density in a semiconductor for a given electric field.

Consider a germanium sample at $T = 300^\circ$K with doping concentrations of $N_d = 0$ and $N_a = 10^{16}$ cm$^{-3}$. Assume complete ionization and assume electron and hole mobilities given in Table 5-1. Calculate the drift current density if the applied electric field is $E = 50$ V/cm.

**Solution:** Since $N_a > N_d$, the semiconductor is p-type and the majority carrier hole concentration, from Chapter 4, is given by

$$p = \frac{N_a - N_d}{2} \left( \frac{N_a - N_d}{2} + n_i^2 \right)^{2} \approx 10^{16} \text{ cm}^{-3}$$

The minority carrier electron concentration is

$$n = \frac{n_i^2}{p} = \frac{2.4 \times 10^{10}}{10^{16}} = 5.76 \times 10^{10} \text{ cm}^{-3}$$

For this extrinsic p-type semiconductor, the drift current density is

$$J_{df} = e(\mu_n n + \mu_p p)E = e\mu_p N_a E$$

Then

$$J_{df} = (1.6 \times 10^{-9})(1900)(10^{16})(50) = 152 \text{ A/cm}^2$$

**Comment:** Significant drift current densities can be obtained in a semiconductor applying relatively small electric fields. We may note from this example that the drift current will usually be due primarily to the majority carrier in an extrinsic semiconductor.

**5.1.2 Mobility Effects**

In the last section, we defined mobility, which relates the average drift velocity of a carrier to the electric field. Electron and hole mobilities are important semiconductor parameters in the characterization of carrier drift, as seen in Equation (5-9).

Equation (5-3) related the acceleration of a hole to a force such as an electric field. We may write this equation as

$$F = m_p \frac{dv}{dt} = eE$$

(5-10)
where $v$ is the velocity of the particle due to the electric field and does not include the random thermal velocity. If we assume that the effective mass and electric field are constants, then we may integrate Equation (5-10) and obtain

$$v = \frac{eE t}{m_p^2}$$  \hspace{1cm} (5-11)

where we have assumed the initial drift velocity to be zero.

Figure 5-1a shows a schematic model of the random thermal velocity and motion of a hole in a semiconductor with zero electric field. There is a mean time between collisions which may be denoted by $\tau_{cp}$. If a small electric field (E-field) is applied as indicated in Figure 5-1b, there will be a net drift of the hole in the direction of the E-field, and the net drift velocity will be a small perturbation on the random thermal velocity, so the time between collisions will not be altered appreciably. If we use the mean time between collisions $\tau_{cp}$ in place of the time $t$ in Equation (5-11), then the mean peak velocity just prior to a collision or scattering event is

$$v_{\text{break}} = \left(\frac{e\tau_{cp}}{m_p^2}\right) E$$  \hspace{1cm} (5-12a)

![Figure 5-1](image-url)  

Figure 5-1 Typical random behavior of a hole in a semiconductor (a) without an electric field and (b) with an electric field.
The average drift velocity is one half the peak value so that we can write

$$\langle v_d \rangle = \frac{1}{2} \left( \frac{e\tau_{cp}}{m_p} \right) E$$  \hspace{1cm} (5-12b)

However, if the proper averaging of the mean time between collisions over the random thermal velocities is performed, the $\frac{1}{2}$ in Equation (5-12b) is eliminated. The hole mobility is then given by

$$\mu_p = \frac{v_d}{\frac{E}{E}} = \frac{e\tau_{cp}}{m_p}$$  \hspace{1cm} (5-13)

The same analysis applies to electrons; thus we can write the electron mobility as

$$\mu_n = \frac{e\tau_{en}}{m_n}$$  \hspace{1cm} (5-14)

where $\tau_{en}$ is the mean time between collisions for an electron.

There are two collision or scattering mechanisms that dominate in a semiconductor and affect the carrier mobility: phonon or lattice scattering, and ionized impurity scattering.

The atoms in a semiconductor crystal have a certain amount of thermal energy at temperatures above absolute zero that causes the atoms to randomly vibrate about their lattice position within the crystal. The lattice vibrations cause a disruption in the perfect periodic potential function we used in the Kronig-Penney model in Chapter 3. A perfect periodic potential in a solid allowed electrons to move unimpeded, or with no scattering, through the crystal. But the thermal vibrations cause a disruption of the potential function, resulting in an interaction between the electrons or holes and the vibrating lattice atoms. This lattice scattering is also referred to as phonon scattering.

Since lattice scattering is related to the thermal motion of atoms, the rate at which the scattering occurs will be a function of temperature. If we denote $\mu_L$ as the mobility that would be observed if only lattice scattering existed, then the scattering theory states that to first order

$$\mu_L \propto T^{-3/2}$$  \hspace{1cm} (5-15)

Mobility that is due to lattice scattering increases as the temperature decreases. Intuitively, we expect the lattice vibrations to decrease as the temperature decreases, which implies that the probability of a scattering event will also decrease, thus increasing mobility.

The temperature dependence of electron and hole mobilities in silicon is shown in Figure 5-2. In lightly doped semiconductors, lattice scattering dominates and the carrier mobility decreases with temperature as we have discussed. The temperature dependence of mobility is proportional to $T^{-n}$. The inserts in the figure show that the parameter $n$ is not equal to $\frac{1}{2}$ as the first order scattering theory predicted. However, mobility does increase as the temperature decreases.
Figure 5-2 (a) Electron and (b) hole mobilities in silicon versus temperature for various doping concentrations. Inserts show temperature dependence for "almost" intrinsic silicon. (From Pierret [4]).
The second interaction mechanism affecting carrier mobility is called ionized impurity scattering. We have seen that impurity atoms are added to the semiconductor to control or alter its characteristics. These impurities are ionized at room temperature so that a coulomb interaction exists between the electrons or holes and the ionized impurities. This coulomb interaction produces scattering or collisions and also alters the velocity characteristics of the charge carrier. If we denote \( \mu_I \) as the mobility that would be observed if only ionized impurity scattering existed, then to first order we have

\[
\mu_I \propto \frac{T^{3/2}}{N_I}
\]  

(5-16)

where \( N_I = N^+_I + N^-_I \) is the total ionized impurity concentration in the semiconductor. If temperature increases, the random thermal velocity of a carrier increases, reducing the time the carrier spends in the vicinity of the ionized impurity center. The less time spent in the vicinity of a coulomb force, the smaller the scattering effect and the larger the expected value of \( \mu_I \). If the number of ionized impurity centers increases, then the probability of a carrier encountering an ionized impurity center increases, implying a smaller value of \( \mu_I \).

Figure 5-3 is a plot of electron and hole mobilities in germanium, silicon, and gallium arsenide at \( T = 300^\circ\text{K} \) as a function of impurity concentration. More accurately, these curves are of mobility versus ionized impurity concentration \( N_I \). As the impurity concentration increases, the number of impurity scattering centers increases, thus reducing mobility.

If \( \tau_L \) is the mean time between collisions due to lattice scattering, then \( dt/\tau_L \) is the probability of a lattice scattering event occurring in a differential time \( dt \). Likewise, if \( \tau_i \) is the mean time between collisions due to ionized impurity scattering, then \( dt/\tau_i \) is the probability of an ionized impurity scattering event occurring in the differential time \( dt \). If these two scattering processes are independent, then the total probability of a scattering event occurring in the differential time \( dt \) is the sum of the individual events, or

\[
\frac{dt}{\tau} = \frac{dt}{\tau_i} + \frac{dt}{\tau_L}
\]  

(5-17)

where \( \tau \) is the mean time between any scattering event.

Comparing Equation (5-17) with the definitions of mobility given by Equation (5-13) or (5-14), we can write

\[
\frac{1}{\mu} = \frac{1}{\mu_i} + \frac{1}{\mu_L}
\]  

(5-18)

where \( \mu_i \) is the mobility due to the ionized impurity scattering process and \( \mu_L \) is the mobility due to the lattice scattering process. The parameter \( \mu \) is the net mobility. With two or more independent scattering mechanisms, the inverse mobilities add, which means that the net mobility decreases.
5.1.3 Conductivity

The drift current density, given by Equation (5-9), may be written as

\[ J_{df} = e(\mu_n n + \mu_p p)E = \sigma E \quad (5-19) \]

where \( \sigma \) is the conductivity of the semiconductor material. The conductivity is given in units of (ohm-cm)\(^{-1} \) and is a function of the electron and hole concentrations and mobilities. We have just seen that the mobilities are functions of impurity concentrations; conductivity, then, is a somewhat complicated function of impurity concentration.

The reciprocal of conductivity is resistivity, which is denoted by \( \rho \) and is given in units of ohm-cm. We can write

\[ \rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)} \quad (5-20) \]

Figure 5-4 is a plot of resistivity as a function of impurity concentration in silicon, germanium, gallium arsenide, and gallium phosphide at \( T = 300^\circ \text{K} \).
Figure 5-4 Resistivity versus impurity concentration at $T = 300\,\text{K}$ in (a) silicon and (b) germanium, gallium arsenide, and gallium phosphide. (From Sze [7]).
Obviously, the curves are not linear functions of \( N_d \) or \( N_a \) due to mobility effects.

If we have a bar of semiconductor material as shown in Figure 5–5 with a voltage applied which produces a current \( I \), then we can write

\[
J = \frac{I}{A}
\]

and

\[
E = \frac{V}{L}
\]

Figure 5–5 Bar of semiconductor material as a resistor.

We can now rewrite Equation (5–19) as

\[
\frac{I}{A} = \sigma \left( \frac{V}{L} \right)
\]

or

\[
V = \frac{I}{\sigma A} L = \frac{\rho L}{\sigma A} I = IR
\]

Equation (5–22b) is Ohm’s law for a semiconductor. The resistance is a function of resistivity, or conductivity, as well as the geometry of the semiconductor.

If we consider, for example, a p-type semiconductor with an acceptor doping \( N_a(N_d = 0) \) in which \( N_a \gg n_i \) and if we assume that the electron and hole mobilities are of the same order of magnitude, then the conductivity becomes

\[
\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p
\]

(5–23)
If we also assume complete ionization, then Equation (5-23) becomes

$$\sigma = e\mu_p N_a \equiv \frac{1}{\rho} \quad (5-24)$$

The conductivity and resistivity of an extrinsic semiconductor are a function primarily of the majority carrier parameters.

We may plot the carrier concentration and conductivity of a semiconductor as a function of temperature for a particular doping concentration. Figure 5-6 shows the electron concentration and conductivity of silicon as a function of inverse temperature for the case when \(N_d = 10^{15} \text{ cm}^{-3}\). In the mid-temperature range, or extrinsic range, as shown, we have complete ionization—the electron concentration remains essentially constant. However, the mobility is a function of temperature so the conductivity varies with temperature in this range. At higher temperatures, the intrinsic carrier concentration increases and begins to dominate the electron concentration as well as the conductivity. In the lower temperature range, freeze-out begins to occur; the electron concentration and conductivity decrease with decreasing temperature.

![Graph of electron concentration and conductivity versus inverse temperature for silicon.](image)

**Figure 5-6** Electron concentration and conductivity versus inverse temperature for silicon. (After Sze [7]).

**Example 5-2**

**Objective:** To determine the majority carrier concentration and mobility given the type and resistivity of a semiconductor.
Consider p-type silicon at \( T = 300^\circ\text{K} \) with a resistivity of \( \rho = 100 \text{ ohm-cm} \). Determine the p-type doping concentration and hole mobility assuming \( N_d = 0 \).

**Solution:** For a p-type semiconductor, the resistivity is

\[
\rho = \frac{1}{e \mu_p \rho} = \frac{1}{e \mu_p N_d}
\]

From Figure 5–4, we may note that a resistivity of 100 ohm-cm corresponds to a low doping concentration. As a first approximation, let \( \mu_p = 480 \text{ cm}^2/\text{V}-\text{sec} \). Then

\[
N_d = \frac{1}{e \mu_p \rho} = \frac{1}{(1.6 \times 10^{-19})(480)(100)} = 1.3 \times 10^{14} \text{ cm}^{-3}
\]

**Comment:** We may note from Figure 5–3 that the hole mobility for this doping concentration is essentially equal to our assumed value. For high-resistivity semiconductors, the mobility corresponding to a low impurity concentration is a good first approximation.

---

**Example 5–3**

**Objective:** To determine the doping concentration and majority carrier mobility given the type and conductivity of a compensated semiconductor.

Consider compensated n-type silicon at \( T = 300^\circ\text{K} \), with a conductivity of \( \sigma = 16 \text{ (ohm-cm)}^{-1} \) and an acceptor doping concentration of \( 10^{17} \text{ cm}^{-3} \). Determine the donor concentration and the electron mobility.

**Solution:** For n-type silicon at \( T = 300^\circ\text{K} \), we can assume complete ionization; therefore the conductivity, assuming \( N_d - N_a \gg n_i \), is given by

\[
\sigma = e \mu_n n = e \mu_n (N_d - N_a)
\]

We have that

\[
16 = (1.6 \times 10^{-19}) \mu_n (N_d - 10^{17})
\]

Since mobility is a function of the ionized impurity concentration, we can use Figure 5–3 along with trial and error to determine \( \mu_n \) and \( N_d \). For example, if we choose \( N_d = 2 \times 10^{17} \), then \( N_f = N_d^+ + N_a^- = 3 \times 10^{17} \) so that \( \mu_n \approx 510 \text{ cm}^2/\text{V}-\text{sec} \) which gives \( \sigma = 8.16 \text{ (ohm-cm)}^{-1} \). If we choose \( N_d = 5 \times 10^{17} \), then \( N_f = 6 \times 10^{17} \) so that \( \mu_n \approx 325 \text{ cm}^2/\text{V}-\text{sec} \) which gives \( \sigma = 20.8 \text{ (ohm-cm)}^{-1} \). The doping is bounded between these two values. Further trial and error yields

\[
N_d = 3.5 \times 10^{17} \text{ cm}^{-3}
\]

and

\[
\mu_n = 400 \text{ cm}^2/\text{V}-\text{sec}
\]

which gives

\[
\sigma \approx 16 \text{ (ohm-cm)}^{-1}
\]

**Comment:** We can see from this example that, in high-conductivity semiconductor material, mobility is a strong function of carrier concentration.
For an intrinsic material, the conductivity can be written as

\[ \sigma_i = e(\mu_n + \mu_p) n_i \]  

(5-25)

The concentrations of electrons and holes are equal in an intrinsic semiconductor, so the intrinsic conductivity includes both the electron and hole mobility. Since, in general, the electron and hole mobilities are not equal, the intrinsic conductivity is not the minimum value possible at a given temperature.

5.1.4 Velocity Saturation

So far in our discussion of drift velocity, we have assumed that mobility is not a function of electric field, meaning that the drift velocity will increase linearly with applied electric field. The total velocity of a particle is the sum of the random thermal velocity and drift velocity. At \( T = 300^\circ \text{K} \), the average random thermal energy is given by

\[ \frac{1}{2} m v_{th}^2 = \frac{1}{2} kT = \frac{1}{2} \left( 0.0259 \right) = 0.03885 \text{ eV} \]  

(5-26)

This energy translates into a mean thermal velocity of approximately \( 10^7 \) cm/sec for an electron in silicon. If we assume an electron mobility of \( \mu_n = 1250 \) cm\(^2\)/V-sec in low-doped silicon, a drift velocity of \( 10^5 \) cm/sec, or 1 percent of the thermal velocity, will be achieved if the applied electric field is approximately 75 V/cm. This applied electric field does not appreciably alter the energy of the electron.

Figure 5-7 is a plot of average drift velocity as a function of applied electric field for electrons and holes in silicon, gallium arsenide, and germanium. At low electric fields where there is a linear variation of velocity with

![Figure 5-7](image-url)
electric field, the slope of the drift velocity versus electric field curve is the mobility. The behavior of the drift velocity of carriers at high electric fields deviates substantially from the linear relationship observed at low fields. The drift velocity of electrons in silicon, for example, saturates at approximately $10^7$ cm/sec at an electric field of approximately 30 kV/cm. If the drift velocity of a charge carrier saturates, then the drift current density will also saturate and become independent of the applied electric field.

The drift velocity versus electric field characteristic of gallium arsenide is more complicated than for silicon or germanium. At low fields, the slope of the drift velocity versus E-field is constant and is the low-field electron mobility, which is approximately 8500 cm$^2$/V-sec for gallium arsenide. The low-field electron mobility in gallium arsenide is much larger than in silicon. As the field increases, the electron drift velocity in gallium arsenide reaches a peak and then decreases. A differential mobility is the slope of the $v_d$ versus $E$ curve at a particular point on the curve and the negative slope of the drift velocity versus electric field represents a negative differential mobility. The negative differential mobility produces a negative differential resistance; this characteristic is used in the design of oscillators.

The negative differential mobility can be understood by considering the $E$ versus $k$ diagram for gallium arsenide which is shown again in Figure 5–8. The density of states effective mass of the electron in the lower valley is $m^*_e = 0.067 m_0$. The small effective mass leads to a large mobility. As the E-field increases, the energy of the electron increases and the electron can be

![Energy-band structure for gallium arsenide showing the upper valley and lower valley in the conduction band. (From Sze [8]).](image)
scattered into the upper valley, where the density of states effective mass is 0.55 $m_0$. The larger effective mass in the upper valley yields a smaller mobility. This intervalley transfer mechanism results in a decreasing average drift velocity of electrons with electric field, or the negative differential mobility characteristic. We will see in a later chapter how this negative differential mobility is used to make a high-frequency GUNN effect oscillator.

5.2 CARRIER DIFFUSION

There is a second mechanism, in addition to drift, that can induce a current in a semiconductor. We may consider a classic physics example in which a container, as shown in Figure 5–9, is divided into two parts by a membrane.

![Figure 5–9 Container divided by a membrane with gas molecules on one side.](image)

The left side contains gas molecules at a particular temperature and the right side is initially empty. The gas molecules are in continual random thermal motion so that, when the membrane is broken, the gas molecules flow into the right side of the container. Diffusion is the process whereby particles flow from a region of high concentration toward a region of low concentration. If the gas molecules were electrically charged, the net flow of charge would result in a diffusion current.

5.2.1 Diffusion Current Density

To begin to understand the diffusion process in a semiconductor, we will consider a simplified analysis. Assume that an electron concentration varies in one dimension as shown in Figure 5–10. The temperature is assumed to be uniform so that the average thermal velocity of electrons is independent of $x$.

To calculate the current, we will determine the net flow of electrons per unit time per unit area crossing the plane at $x = 0$. If the distance $l$ shown in Figure 5–10 is the mean-free path of an electron, that is, the average distance an electron travels between collisions ($l = \nu_{th}/\tau_{tr}$), then on the average, electrons moving to the right at $x = -l$ and electrons moving to the left at $x = +l$ will cross the $x = 0$ plane. One half of the electrons at $x = -l$ will be traveling to the right at any instant of time and one half of the electrons at $x = +l$ will be traveling to the left at any given time. The net rate of electron flow, $F_x$, in the $+x$ direction at $x = 0$ is given by
\[ F_n = \frac{1}{2} n(-l)v_{th} - \frac{1}{2} n(+l)v_{th} = \frac{1}{2} v_{th}[n(-l) - n(+l)] \]  \hspace{1cm} (5-27)

If we expand the electron concentration in a Taylor series about \( x = 0 \) keeping only the first two terms, then Equation (5-27) can be written as

\[ F_n = \frac{1}{2} v_{th} \left( [n(0) - l \frac{dn}{dx}] - [n(0) + l \frac{dn}{dx}] \right) \]  \hspace{1cm} (5-28)

which becomes

\[ F_n = -v_{th}l \frac{dn}{dx} \]  \hspace{1cm} (5-29)

Each electron has a charge \((-e)\) so the current is

\[ J = -eF_n = +ev_{th}l \frac{dn}{dx} \]  \hspace{1cm} (5-30)

The current described by Equation (5-30) is the electron diffusion current and is proportional to the spatial derivative, or density gradient, of the electron concentration.

The diffusion of electrons from a region of high concentration to a region of low concentration produces a flux of electrons flowing in the negative \( x \) direction for this example. Since electrons have a negative charge, the conventional current direction is in the positive \( x \) direction. These one-dimensional flux and current directions are shown in Figure 5-11a. We may write the electron diffusion current density for this one-dimensional case, in the form

\[ J_{\text{rel,df}} = eD_n \frac{dn}{dx} \]  \hspace{1cm} (5-31)
where $D_n$ is called the electron diffusion coefficient, has units of cm$^2$/sec, and is a positive quantity. If the electron density gradient becomes negative, the electron diffusion current density will be in the negative $x$ direction.

Figure 5-11b shows an example of a hole concentration as a function of distance in a semiconductor. The diffusion of holes, from a region of high concentration to a region of low concentration, produces a flux of holes in the negative $x$ direction. Since holes are positively charged particles, the conventional diffusion current density is also in the negative $x$ direction. The hole diffusion current density is proportional to the hole density gradient and to the electronic charge, so we may write

$$J_{p\text{diff}} = -eD_p \frac{dp}{dx} \quad (5-32)$$

for the one-dimensional case. The parameter $D_p$ is called the hole diffusion coefficient, has units of cm$^2$/sec, and is a positive quantity. If the hole density gradient becomes negative, the hole diffusion current density will be in the positive $x$ direction.
Example 5-4

Objective: To calculate the diffusion current density given a density gradient.

Assume that, in an n-type gallium arsenide semiconductor at \( T = 300^\circ \text{K} \), the electron concentration varies linearly from \( 1 \times 10^{18} \) to \( 7 \times 10^{17} \) cm\(^{-3} \) over a distance of 0.10 cm. Calculate the diffusion current density if the electron diffusion coefficient is \( D_n = 225 \) cm\(^2\)/sec.

Solution: The diffusion current density is given by

\[
J_{\text{diff}} = eD_n \frac{dn}{dx} eD_n \frac{\Delta n}{\Delta x}
\]

\[
= (1.6 \times 10^{-19})(225) \left( \frac{1 \times 10^{18} - 7 \times 10^{17}}{0.10} \right) = 108 \text{ A/cm}^2
\]

Comment: A significant diffusion current density can be generated in a semiconductor material with only a modest density gradient.

5.2.2 Total Current Density

We now have four possible independent current mechanisms in a semiconductor. These components are electron drift and diffusion currents and hole drift and diffusion currents. The total current density is the sum of these four components, or for the one-dimensional case, we have

\[
J = e\mu_n E_x + e\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx} \quad (5-33)
\]

This equation may be generalized to three dimensions as

\[
J = e\mu_n E + e\mu_p E + eD_n \nabla n - eD_p \nabla p \quad (5-34)
\]

The electron mobility gives an indication of how well an electron moves in a semiconductor due to the force of an electric field. The electron diffusion coefficient gives an indication of how well an electron moves in a semiconductor as a result of a density gradient. The electron mobility and diffusion coefficient are not independent parameters. Similarly, the hole mobility and diffusion coefficient are not independent parameters. The relationship between mobility and the diffusion coefficient will be developed in the next section.

The expression for the total current in a semiconductor contains four terms. Fortunately in most situations, we will only need to consider one term at any one time at a particular point in a semiconductor.
5.3 GRADED IMPURITY DISTRIBUTION

In most cases so far, we have assumed that the semiconductor is uniformly doped. In many semiconductor devices, however, there may be regions that are nonuniformly doped. We will investigate how a nonuniformly doped semiconductor reaches thermal equilibrium and, from this analysis, we will derive the Einstein relation which relates mobility and the diffusion coefficient.

5.3.1 Induced Electric Field

Consider a semiconductor that is nonuniformly doped with donor impurity atoms. If the semiconductor is in thermal equilibrium, the Fermi energy level is constant through the crystal so the energy-band diagram may qualitatively look like that shown in Figure 5–12. The doping concentration decreases as \( x \) increases in this case. There will be a diffusion of majority carrier electrons from the region of high concentration to the region of low concentration which is in the \( +x \) direction. The flow of negative electrons leaves behind positively charged donor ions. The separation of positive and negative charge induces an electric field that is in a direction to oppose the diffusion process. When equilibrium is reached, the mobile carrier concentration is not exactly equal to the fixed impurity concentration and the induced electric field prevents any further separation of charge. In most cases of interest, the space charge induced by this diffusion process is a small fraction of the impurity concentration, thus the mobile carrier concentration is not too different from the impurity dopant density.

The electric potential \( \phi \) is related to electron potential energy by the charge \((-e)\), so we can write

\[
\phi = + \frac{1}{e} (E_F - E_F)
\]  

(5–35)

Figure 6–12  Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.
The electric field for the one-dimensional situation is defined as

\[ E_x = -\frac{dB}{dx} = \frac{1}{e} \frac{dE_{EI}}{dx} \]  \hspace{1cm} (5-36)

If the intrinsic Fermi level changes as a function of distance through a semiconductor in thermal equilibrium, an electric field exists in the semiconductor.

If we assume a quasi-neutrality condition in which the electron concentration is almost equal to the donor impurity concentration, then we can still write

\[ n_0 = n_i \exp \left( \frac{E_F - E_{FI}}{kT} \right) \approx N_d(x) \]  \hspace{1cm} (5-37)

Solving for \( E_F - E_{FI} \), we obtain

\[ E_F - E_{FI} = kT \ln \left( \frac{N_d(x)}{n_i} \right) \]  \hspace{1cm} (5-38)

The Fermi level is constant for thermal equilibrium so when we take the derivative with respect to \( x \) we obtain

\[ -\frac{dE_{EI}}{dx} = \frac{kT}{N_d(x)} \frac{dN_d(x)}{dx} \]  \hspace{1cm} (5-39)

The electric field can then be written, combining Equations (5-39) and (5-36), as

\[ E_x = -\left( \frac{kT}{e} \right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx} \]  \hspace{1cm} (5-40)

Since we have an electric field, there will be a potential difference through the semiconductor due to the nonuniform doping.

---

**Example 5-5**

**Objective:** To determine the induced electric field in a semiconductor in thermal equilibrium, given a linear variation in doping concentration.

Assume that the donor concentration in an n-type semiconductor at \( T = 300^\circ K \) is given by

\[ N_d(x) = 10^{16} - 10^{9}x \] \( \text{(cm}^{-3}\text{)} \)

where \( x \) is given in cm and ranges between \( 0 \leq x \leq 1 \mu m \).

**Solution:** Taking the derivative of the donor concentration, we have

\[ \frac{dN_d(x)}{dx} = -10^{9} \] \( \text{(cm}^{-4}\text{)} \)

The electric field is given by Equation (5-40) so that we have
\[ E_x = \frac{-(0.0259)(-10^9)}{(10^{16} - 10^{19})} \]

At \( x = 0 \), for example, we find
\[ E_x = 25.9 \text{ V/cm} \]

**Comment:** We may recall from our previous discussion of drift current that fairly small electric fields can produce significant drift current densities, so that an induced electric field from nonuniform doping can significantly influence semiconductor device characteristics.

### 5.3.2 The Einstein Relation

If we consider the nonuniformly doped semiconductor represented by the energy-band diagram shown in Figure 5-12 and assume there are no electrical connections so that the semiconductor is in thermal equilibrium, then the individual electron and hole currents must be zero. We can write

\[ J_n = 0 = e\mu_n E_x + eD_n \frac{dn}{dx} \quad (5-41) \]

If we assume quasi-neutrality so that \( n \equiv N_d(x) \), then Equation (5-41) can be rewritten as

\[ J_n = 0 = e\mu_n N_d(x) E_x + eD_n \frac{dN_d(x)}{dx} \quad (5-42) \]

Substituting the expression for the electric field from Equation (5-40) into Equation (5-42), we obtain

\[ 0 = -e\mu_n N_d(x) \left( \frac{kT}{e} \right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx} \quad (5-43) \]

Equation (5-43) is valid for the condition

\[ \frac{D_n}{\mu_n} = \frac{kT}{e} \quad (5-44a) \]

The hole current must also be zero in the semiconductor. From this condition, we can show that

\[ \frac{D_p}{\mu_p} = \frac{kT}{e} \quad (5-44b) \]

Combining Equations (5-44a) and (5-44b), we have

\[ \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e} \quad (5-45) \]

The diffusion coefficient and mobility are not independent parameters. This
relation between the mobility and diffusion coefficient, given by Equation (5–45), is known as the Einstein relation.

Example 5–6

Objective: To determine the diffusion coefficient given the carrier mobility.
Assume that the mobility of a particular carrier is 1000 cm²/V-sec at T = 300°K.

Solution: Using the Einstein relation, we have that

\[ D = \left( \frac{kT}{e} \right) \mu = (0.0259)(1000) = 25.9 \text{ cm}^2/\text{sec} \]

Comment: Although this example is fairly simple and straightforward, it is important to keep in mind the relative order of magnitudes of the mobility and diffusion coefficient. The diffusion coefficient is approximately 40 times smaller than the mobility at room temperature.

Table 5–2 shows the diffusion coefficient values at T = 300°K corresponding to the mobilities listed in Table 5–1 for silicon, gallium arsenide, and germanium.

<table>
<thead>
<tr>
<th>Material</th>
<th>μ_n</th>
<th>D_n</th>
<th>μ_p</th>
<th>D_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>1350</td>
<td>35</td>
<td>480</td>
<td>12.4</td>
</tr>
<tr>
<td>Gallium arsenide</td>
<td>8500</td>
<td>220</td>
<td>400</td>
<td>10.4</td>
</tr>
<tr>
<td>Germanium</td>
<td>3900</td>
<td>101</td>
<td>1900</td>
<td>49.2</td>
</tr>
</tbody>
</table>

The relation between the mobility and diffusion coefficient given by Equation (5–45) contains temperature. It is important to keep in mind that the major temperature effects are a result of lattice scattering and ionized impurity scattering processes, as discussed in Section 5.1.2. As the mobilities are strong functions of temperature due to the scattering processes, the diffusion coefficients are also strong functions of temperature. The specific temperature dependence given in Equation (5–45) is a small fraction of the real temperature characteristic.

5.4 THE HALL EFFECT

The Hall effect is a consequence of the forces that are exerted on moving charges by electric and magnetic fields. The Hall effect is used to distinguish whether a semiconductor is n-type or p-type¹ and is used to measure the

¹ We will assume an extrinsic semiconductor material in which the majority carrier concentration is much larger than the minority carrier concentration.
majority carrier concentration and majority carrier mobility. The Hall effect device, as discussed in this section, is used to experimentally measure semiconductor parameters. However, the Hall effect device is used extensively in engineering applications as a magnetic probe and in other circuit applications.

The force on a particle having a charge $q$ and moving in a magnetic field is given by

$$F = qv \times B$$

where the cross product is taken between velocity and magnetic field so that the force vector is perpendicular to both the velocity and magnetic field.

Figure 5-13 illustrates the Hall effect. A semiconductor with a current $I_x$ is placed in a magnetic field perpendicular to the current. In this case, the magnetic field is in the $z$-direction. Electrons and holes flowing in the semiconductor will experience a force as indicated in the figure. The force on both electrons and holes is in the ($-y$) direction. In a p-type semiconductor ($p_0 > n_0$), there will be a buildup of positive charge on the $y = 0$ surface of the semiconductor and, in an n-type semiconductor ($n_0 > p_0$), there will be a buildup of negative charge on the $y = 0$ surface. This net charge induces an electric field in the $y$-direction as shown in the figure. In steady state, the magnetic field force will be exactly balanced by the induced electric field.

Figure 5-13  Geometry for measuring the Hall effect.
force. This balance may be written as

\[ F = q(E + v \times B) = 0 \]  
(5.47a)

which becomes

\[ qE_y = qv_x B_z \]  
(5.47b)

The induced electric field in the y-direction is called the Hall field. The Hall field produces a voltage across the semiconductor which is called the Hall voltage. We can write

\[ V_H = +E_H W \]  
(5-48)

where \(E_H\) is assumed positive in the +y direction and \(V_H\) is positive with the polarity shown.

In a p-type semiconductor in which holes are the majority carrier, the Hall voltage will be positive as defined in Figure 5–13. In an n-type semiconductor in which electrons are the majority carrier, the Hall voltage will have the opposite polarity. The polarity of the Hall voltage is used to determine whether an extrinsic semiconductor is n-type or p-type.

Substituting Equation (5–48) into Equation (5–47), we have

\[ V_H = v_x W B_z \]  
(5–49)

For a p-type semiconductor, the drift velocity of holes can be written as

\[ v_{dc} = \frac{J_z}{e \rho} = \frac{I_z}{(e \rho)(W d)} \]  
(5–50)

where \(e\) is the magnitude of the electronic charge. Combining Equations (5–50) and (5–49), we have

\[ V_H = \frac{I_z B_z}{e d \rho} \]  
(5–51)

or, solving for the hole concentration, we obtain

\[ \rho = \frac{I_z B_z}{e d V_H} \]  
(5–52)

The majority carrier hole concentration is determined from the current, magnetic field, and Hall voltage.

For an n-type semiconductor, the Hall voltage is given by

\[ V_H = -\frac{I_z B_z}{n e d} \]  
(5–53)

so that the electron concentration is

\[ n = -\frac{I_z B_z}{e d V_H} \]  
(5–54)

We may note that the Hall voltage is negative for the n-type semiconductor;
therefore, the electron concentration determined from Equation (5–54) is actually a positive quantity.

Once the majority carrier concentration has been determined, we can calculate the low-field majority carrier mobility. For a p-type semiconductor, we can write

\[ J_x = e \mu_p V_x \]  \hspace{1cm} (5–55)

The current density and electric field can be converted to current and voltage so that Equation (5–55) becomes

\[ \frac{I_x}{Wd} = \frac{e \mu_p V_x}{L} \]  \hspace{1cm} (5–56)

The hole mobility is then given by

\[ \mu_p = \frac{I_x L}{e V_x Wd} \]  \hspace{1cm} (5–57)

Similarly for an n-type semiconductor, the low-field electron mobility is determined from

\[ \mu_n = \frac{I_x L}{e n V_x Wd} \]  \hspace{1cm} (5–58)

---

**Example 5–7**

**Objective:** To determine the majority carrier concentration and mobility given Hall effect parameters.

Consider the geometry shown in Figure 5–13. Let \( L = 10^{-1} \text{ cm} \), \( W = 10^{-2} \text{ cm} \), and \( d = 10^{-2} \text{ cm} \). Also assume that \( I_x = 1.0 \text{ mA} \), \( V_x = 12.5 \) volts, \( B_x = 500 \) gauss = \( 5 \times 10^{-2} \) tesla, and \( V_H = -6.25 \) mV.

**Solution:** A negative Hall voltage for this geometry implies that we have an n-type semiconductor. Using Equation (5–54), we can calculate the electron concentration as

\[ n = \frac{-\left(10^{-3}\right)\left(5 \times 10^{-2}\right)}{\left(1.6 \times 10^{-19}\right)\left(10^{-5}\right)\left(-6.25 \times 10^{-3}\right)} = 5 \times 10^{21} \text{ m}^{-3} = 5 \times 10^{15} \text{ cm}^{-3} \]

The electron mobility is then determined from Equation (5–58) as

\[ \mu_n = \frac{(10^{-3})(10^{-1})}{\left(1.6 \times 10^{-19}\right)\left(5 \times 10^{21}\right)\left(12.5\right)\left(10^{-5}\right)\left(10^{-5}\right)} = 0.10 \text{ m}^2/\text{V-sec} \]

or

\[ \mu_n = 1000 \text{ cm}^2/\text{V-sec} \]

**Comment:** It is important to note that the MKS units must be used consistently in the Hall effect equations to yield correct results.
5.5 SUMMARY AND REVIEW

In this chapter we have considered carrier transport, the process by which charge moves in a semiconductor to produce current. We explored the theory of the two transport mechanisms, drift and diffusion.

We first considered drift. The motion or drift of electrons and holes due to electric fields leads to drift currents. The drift current density in a semiconductor is a function of the concentration of electrons and holes and is also a function of the average drift velocity of these charge carriers. The net flow of electrons and holes, due to an applied electric field, is in opposite directions because of the difference in charge, but the drift currents generated by the electrons and holes are in the same direction as the electric field.

The average drift velocity of a carrier is a function of the electric field and carrier mobility. Carrier mobility is an indicator of how well an electron or hole will drift through the semiconductor, and a function of the scattering or collision processes within the semiconductor. Two scattering mechanisms are lattice scattering and impurity scattering. Lattice scattering occurs because atoms in a semiconductor contain a certain amount of thermal energy in the form of atomic vibrations. The interatomic distance between atoms is then fluctuating, the perfect periodic potential function is altered, and the motion of the carrier through the semiconductor is disrupted. Since this scattering mechanism is a function of the thermal motion of atoms, the associated component of mobility is a strong function of temperature. Mobility decreases as temperature increases. Impurity scattering occurs because donor and acceptor impurities are ionized: a coulomb interaction occurs between the charged impurities and electrons and holes. When carriers drift into the vicinity of a charged impurity atom, they will be deflected or scattered. Mobility decreases as the impurity concentration increases.

Conductivity and resistivity are parameters used extensively in semiconductor material specifications, resistivity being the reciprocal of conductivity. These parameters are functions of electron and hole concentrations and mobilities.

We then considered diffusion. Diffusion of electrons and holes in a semiconductor leads to diffusion currents proportional to the gradients in electron and hole concentrations. The constant of proportionality is the electron diffusion coefficient or hole diffusion coefficient. The diffusion coefficients indicate how well the charge carriers diffuse in a semiconductor. The carrier diffusion coefficient and mobility are not independent parameters, but are related by the Einstein relation.

The Hall effect is used to experimentally determine conductivity type, majority carrier concentration, and majority carrier mobility. A magnetic field is applied perpendicular to a current in the semiconductor. A moving charge particle in a magnetic field experiences a force perpendicular to both the magnetic field and direction of motion. An electric field, the Hall field, is induced in the semiconductor due to these forces. The induced field pro-
duces a voltage, the Hall voltage, across the semiconductor. The polarity of the Hall voltage is a function of the semiconductor conductivity type and the magnitude of the Hall voltage is inversely proportional to the majority carrier concentration.

The two current processes resulting from drift and diffusion in a semiconductor are the basis for the current-voltage characteristics of semiconductor devices. We have implicitly assumed in these discussions that, though there is a net flow of electrons and holes, thermal equilibrium has not been substantially disturbed. In the next chapter, we will consider nonequilibrium processes in a semiconductor and then apply these two transport processes to both the equilibrium and nonequilibrium carriers in semiconductor devices.

GLOSSARY OF IMPORTANT TERMS

Conductivity: A material parameter related to carrier drift; quantitatively, the ratio of drift current density to electric field.

Diffusion: The process whereby particles flow from a region of high concentration to a region of low concentration.

Diffusion coefficient: The parameter relating particle flux to the particle density gradient.

Diffusion current: The current that results from the diffusion of charged particles.

Drift: The process whereby charged particles move while under the influence of an electric field.

Drift current: The current that results from the drift of charged particles.

Drift velocity: The average velocity of charged particles in the presence of an electric field.

Einstein relation: The relation between the mobility and the diffusion coefficient.

Hall voltage: The voltage induced across a semiconductor in a Hall-effect measurement.

Ionized impurity scattering: The interaction between a charged carrier and an ionized impurity center.

Lattice scattering: The interaction between a charged carrier and a thermally vibrating lattice atom.

Mobility: The parameter relating carrier drift velocity and electric field.

Resistance: The reciprocal of conductivity; a material parameter that is a measure of the resistance to current.

Velocity saturation: The saturation of carrier drift velocity with increasing electric field.