This experimental transistor, shown in a scanning electron microscope image and made by scientists of the IBM Research Division, is the world's smallest transistor: the ovals you see are only 150 nm long. Today's memory chips have a capacity of 16 megabits. With this new transistor, memory chips may soon have a capacity of 4 gigabits or even more, which will dramatically increase the capabilities of computers of all sizes. But what is a transistor and how does it function?
46-1 THE PROPERTIES OF SOLIDS

We have seen how well quantum theory works when we apply it to individual atoms. In this chapter we hope to show, by a single broad example, that this powerful theory works just as well when we apply it to aggregates of atoms in the form of solids.

Every solid has an enormous range of properties that we can choose to examine. Is it transparent? Can you hammer it out into a flat sheet? What kinds of waves travel through it and at what speeds? Does it have interesting magnetic properties? Is it a good heat conductor? What is its crystal surface? Does it have special surface properties? . . . The list goes on. We choose here to focus on a single question: "What are the mechanisms by which a solid conducts, or does not conduct, electricity?" As we shall see, the laws that govern electrical conduction are quantum laws.

46-2 ELECTRICAL CONDUCTIVITY

In studying electrical conductivity, we choose to examine only solids whose atoms are arranged to form a periodic three-dimensional lattice. Figure 46-1 shows such lattice structures for carbon (in the form of diamond), silicon, and copper. We shall not consider such materials as plastic, glass, or rubber, whose atoms are not arranged in any such regular way.

The basic electrical measurement that we can make on a sample is its electrical resistivity \( \rho \) at room temperature; see Section 28-4. By measuring \( \rho \) at various temperatures, we can also obtain a value for \( \alpha \), the temperature coefficient of resistivity. Finally, by making Hall effect measurements (see Section 30-4) we can find a value for \( n \), the number of charge carriers per unit volume in the material being tested.

From measurements of the room temperature resistivity alone, we quickly discover that there are some materials—we call them insulators—that for all practical purposes do not conduct electricity at all. Diamond, for example, has a resistivity of about \( 10^{18} \, \Omega \cdot m \), greater than that of copper by a factor of about \( 10^{24} \).

We can use our measured values of \( \rho \), \( \alpha \), and \( n \) to divide most noninsulators into two major categories: metals, such as copper, and semiconductors, such as silicon. As we see from Table 46-1, a typical semiconductor (silicon), compared with a typical conductor (copper), (1) has far fewer charge carriers, (2) has a considerably larger resistivity, and (3) has a temperature coefficient of resistivity that is both large and negative. That is, although the resistivity of a metal increases with temperature, that of a semiconductor decreases.

We have now established an experimental basis for framing our central question about the conduction of electricity in solids—"What are the mechanisms by which a solid conducts, or does not conduct, electricity?"

![Figure 46-1](a) The crystal structures for carbon (in the form of diamond) and for silicon, which happen to be identical. In this structure, as the darkened spheres show, each atom is bonded to four of its neighbors. (b) The crystal structure for copper, an arrangement called face-centered cubic.

### Table 46-1

<table>
<thead>
<tr>
<th>Type of conductor</th>
<th>UNIT</th>
<th>COPPER</th>
<th>SILICON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of charge carriers, ( \times 10^{28} ) m(^{-3} )</td>
<td>Metal</td>
<td>( 9 \times 10^{28} )</td>
<td>( 1 \times 10^{15} )</td>
</tr>
<tr>
<td>Resistivity, ( \rho )</td>
<td>( \Omega \cdot m )</td>
<td>( 2 \times 10^{-8} )</td>
<td>( 3 \times 10^{3} )</td>
</tr>
<tr>
<td>Temperature coefficient of resistivity, ( \alpha )</td>
<td>K(^{-1} )</td>
<td>(+ 4 \times 10^{-3})</td>
<td>(- 70 \times 10^{-3})</td>
</tr>
</tbody>
</table>

*All values are for room temperature.*

*The value for the semiconductor includes both electrons and holes.*
tion of electricity in solids. We pose it in specific terms:

What is there about diamond that makes it an insulator, about copper that makes it a metal, and about silicon that makes it a semiconductor?

As we shall see, quantum physics provides the answers to this question.

46-3 ENERGY LEVELS IN A SOLID

The distance between adjacent copper atoms in solid copper is 260 pm. Consider, as in Fig. 46-2a, two copper atoms that are separated by a much greater distance than this. As Fig. 46-2b shows, each of these isolated atoms has associated with it an array of discrete quantum states, each state defined by its unique set of quantum numbers. In the ground state of the neutral copper atom, its 29 electrons occupy the 29 states of this array that are lowest in energy, each state containing but a single electron as the Pauli exclusion principle requires.

If we bring the atoms of Fig. 46-2a closer together, they will—speaking loosely—gradually begin to sense each other’s presence. In the formal language of quantum physics, their wave functions will begin to overlap. This overlap will occur first for the wave functions of the valence electrons, which, because they spend most of their time in the outer regions of the electron cloud of the isolated atom, are the first to make contact.

When the wave functions overlap, we no longer speak of two independent and isolated systems but of a single two-atom system containing $2 \times 29 = 58$ electrons. The Pauli principle requires that each of these electrons must occupy a different quantum state. The only way that this can happen is for each energy level of the isolated atom to split into two levels for the two-atom system.

We can bring up further atoms and in this way gradually construct a lattice of solid copper. If our specimen contains $N$ atoms, each level of the isolated copper atom must be split into $N$ levels. In this way, each level of the isolated atom becomes a band of levels in the solid. In a typical solid, an energy band is a few electron-volts wide. Since $N$ is of the order of the Avogadro number, we can see that the individual energy levels within a band are very close together indeed.

Figure 46-3 suggests the band structure of the levels in a hypothetical solid in which we have assumed, for simplicity, that the bands do not overlap. The gaps between the bands represent ranges of energy that no electron may possess. In much the same way...
way, electrons in individual atoms cannot possess energies that lie between the discrete allowed levels of the atom.

Note in Fig. 46-3 that the bands that lie lower in energy are narrower than those that lie higher. This is because these low-energy bands correspond to levels in the isolated atom that are occupied by electrons that spend most of their time deep within the electron cloud of the atom. The wave functions of these core electrons thus do not overlap as much as do the wave functions of the outer, or valence, electrons and for that reason the splitting of the levels —although it must occur—is not as great as it is for the levels normally occupied by the outer electrons.

Now that we have established the pattern of levels for a solid, we are ready to consider how these levels are filled with electrons. We shall see how this will lead us in a convincing way to the answers to the question that we raised at the end of Section 46-2.

### 46-4 INSULATORS

The feature that defines an **insulator** is that, as Fig. 46-4 shows, the highest occupied level coincides with the top of a band. In addition, this band must be separated from the unoccupied band above it by a substantial energy gap $E_g$. For diamond, $E_g = 5.4 \text{ eV}$, a value about 140 times larger than the average thermal energy of a free particle at room temperature.

By definition, an insulator is a solid through which electrons cannot flow as a directed drift current. Let us see why. If you apply an electric field $\mathbf{E}$ to an insulator, it will exert a force $-e\mathbf{E}$ on each electron. Classically, this force will cause the electron to increase the component of its velocity in the direction $-\mathbf{E}$, which in turn means that its kinetic energy will change. In quantum terms, if the energy of an electron changes, the electron must move to a different energy level within the solid. In an insulator, however, the Pauli principle prevents the electron from doing so because all other levels within the band into which the electron might move are already occupied. These electrons are in total gridlock. It is as if a child tries to climb a ladder on which other children are standing, one every rung; since there are no vacant rungs, no one can move.

There are plenty of vacant levels in the band above the filled band in Fig. 46-4 but, if an electron is to occupy one of these levels, it must somehow jump across the gap that separates the two bands. It cannot pause at a way station within the gap because all energies in this range are strictly forbidden. In diamond, the gap is simply too wide for any detectable number of electrons to make it to the vacant band, either by the action of an external electric field or by thermal agitation.

### 46-5 METALS: QUALITATIVE

The feature that defines a **metal** is that, as Fig. 46-5 shows, the highest occupied level (at the absolute zero of temperature) falls somewhere in the middle of a band. The electrons that occupy this partially filled band are the valence electrons of the atoms.

Only in metals is there an energy gap between the filled band and the next highest vacant band. This energy gap is often referred to as the **band gap**. The conduction electrons of the metal occupy the highest partially filled band. Note that vacant levels are available within the band so that these electrons can change their energies and conduction can take place. Lower lying bands are completely filled by the core electrons, that is, those electrons held close to the lattice sites and not free to move through the solid.

What happens at absolute zero? The electron density falls far below the density of the solid. It is clear that the highest band of Fig. 46-5 is not completely filled. Indeed, thermal agitation is sufficient to fill the Fermi level with the maximum number of electrons (at a fluid state).

**FIGURE 46-5** An idealized representation of the bandgap pattern for a metal at the absolute zero of temperature. The conduction electrons of the metal occupy the highest partially filled band. Note that vacant levels are available within the band so that these electrons can change their energies and conduction can take place. Lower lying bands are completely filled by the core electrons, that is, those electrons held close to the lattice sites and not free to move through the solid.
will cause the electron by the random thermal jiggling of the lattice. At $T = 1000 \text{ K}$, $kT = 0.086 \text{ eV}$; no electron can hope to have its energy changed by more than a few times this relatively small amount by thermal agitation alone. All the "action" takes place for electrons whose energies are close to the Fermi energy. It has been said, somewhat poetically, that thermal agitation normally causes only ripples on the surface of the Fermi sea; the vast depths of that sea lie undisturbed.

**Electrical Conduction in a Metal**

If you apply an electric field $E$ to a metal, the field exerts a force $-eE$ on each electron. This force, during time $\Delta t$, causes every conduction electron in the metal to acquire a velocity increment $\Delta v$ in the direction of $-E$. This change in velocity requires that the electrons change their energies, but there are vacant levels available so that these rearrangements can be made. To return to our previous metaphor, there are now vacant rungs on the upper half of the ladder.

The velocities of the individual conduction electrons do not increase without limit, however, because of collisions associated with the thermal vibrations of the lattice. Thus, after a certain time $\tau$, called the relaxation time, the drift velocity of the conduction electrons settles down to a constant limiting value, which we associate with the constant current that is set up by the applied electric field. Note that, although all the conduction electrons contribute to the current, only electrons close to the Fermi energy are able to make collisions and thus play their role in establishing the limiting value of the drift velocity. It is only these electrons that have ample vacant levels nearby into which they can move after they have experienced a scattering event.

In Section 28-6, we presented the following equation for the resistivity of a metallic conductor,

$$\rho = \frac{m}{nee^2\tau}, \quad (46-1)$$

in which $m$ is the mass of the electron, $e$ is its charge, and $n$ is the number density of the charge carriers, that is, the number of conduction electrons per unit volume. Although we derived this equation on a classical basis, it holds true when the quantization of the electron energy is taken into account. The quantity $\tau$ is the relaxation time to which we referred in the preceding paragraph.
CHAPTER 46
CONDUCTION OF ELECTRICITY IN SOLIDS

SAMPLE PROBLEM 46-1

How many conduction electrons are there in a copper cube 1.00 cm on edge?

**SOLUTION** In copper, there is one conduction electron per atom so that \( N \) is given by

\[
N = na^3,
\]

in which \( n \) is the number of atoms per unit volume and \( a \) (= 1.00 cm) is the length of the cube edge. We can find \( n \) from

\[
n = \frac{N_A d}{A}
\]

in which \( N_A \) is the Avogadro constant, \( A \) is the molar mass of copper, and \( d \) is the density of copper. If we substitute values for these quantities, we find

\[
n = \frac{(6.02 \times 10^{23} \text{ atoms/mol})(8900 \text{ kg/m}^3)}{0.6537 \text{ kg/mol}} = 8.43 \times 10^{28} \text{ atoms/m}^3.
\]

\[
= 8.43 \times 10^{28} \text{ electrons/m}^3.
\]

From Eq. 46-2 we then have

\[
N = na^3 = (8.43 \times 10^{28} \text{ electrons/m}^3)(1.00 \times 10^{-2} \text{ m})^3 = 8.43 \times 10^{22} \text{ electrons}. \quad \text{(Answer)}
\]

The Pauli exclusion principle requires that each of these electrons occupy a different quantum state. These states are distributed over an energy interval of only 7.0 eV (the Fermi energy) so that the average spacing between them is very small indeed.

Not all these states have different energies. Consider, for example, an electron moving along the \( x \) axis with speed \( v \). It has the same energy as an electron moving with this same speed in any other direction in the solid but, because its motion is different, it is in a different quantum state, described by a different wave function.

**SAMPLE PROBLEM 46-2**

a. What is the speed of a conduction electron in copper with a kinetic energy equal to the Fermi energy (= 7.0 eV)?

**SOLUTION** The total energy \( E \) of the conduction electrons is all kinetic and we can write, if \( E = E_F \),

\[
E_F = \frac{1}{2}mv^2,
\]

in which \( v_F \) is the Fermi speed. Solving for \( v_F \) yields

\[
v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\left(\frac{2}{7}(7.0 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})\right)} \approx 9.11 \times 10^{-31} \text{ kg}
\]

\[
= 1.6 \times 10^6 \text{ m/s}. \quad \text{(Answer)}
\]

You must not confuse this speed with the drift speed of the conduction electrons, which is typically 10^{-5} m/s and is thus smaller by about a factor of 10^{11}. As we explained more fully in Section 28-9, the drift speed is the average speed at which electrons actually drift through a conductor when an electric field is applied; the Fermi speed is the average speed of these electrons between collisions.

b. What is the average time \( \tau \) between collisions for the conduction electrons in copper? The resistivity of copper at room temperature is 1.7 \times 10^{-8} \Omega \cdot m.

**SOLUTION** Solving Eq. 46-1 for \( \tau \) yields

\[
\tau = \frac{m}{ne^2\rho} = \frac{9.11 \times 10^{-31} \text{ kg}}{8.45 \times 10^{26} \text{ m}^{-3} \times \frac{1}{(1.6 \times 10^{-19} \text{ C})^2 \times (1.7 \times 10^{-8} \Omega \cdot \text{m})}} = 2.5 \times 10^{-11} \text{ s}. \quad \text{(Answer)}
\]

c. What mean free path \( \lambda \) may be calculated from the results of (a) and (b) above?

**SOLUTION** We have

\[
\lambda = \frac{v_F \tau}{(1.6 \times 10^6 \text{ m/s})(2.5 \times 10^{-14} \text{ s}) = 4.0 \times 10^{-8} \text{ m} = 40 \text{ nm}. \quad \text{(Answer)}
\]

In the copper lattice the centers of neighboring atoms are 0.26 nm apart. Thus a typical conduction electron can move a substantial distance, about 150 interatomic distances, through a copper lattice at room temperature before making a collision.

**46-6 METALS: QUANTITATIVE**

Now let us look at the conduction of electricity in a metal quantitatively, under several headings.

**Counting the Quantum States**

We start by counting the number of distinct quantum states in the partially filled band of Fig. 46-5. We
The resistivity \( \rho \) of Cu is typically \( 1.7 \times 10^{-8} \Omega \cdot m \) \((\text{Answer})\). As between collisions for copper? The resistivity of \( 7 \times 10^{-8} \Omega \cdot m \) \( \tau \) yields \( 2.5 \times 10^{-14} \text{s} \) \((\text{Answer})\).

- Paragraphs discussing collision times, resistivity, and drift speed.

**Quantum States**

- Number of distinct quantum states in a metal.

**Antidote**

- Natural language explanation of quantum states and energy bands.
Filling the States for $T > 0$

It can be shown that the probability function for $T > 0$ is given by

$$
\rho(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{(probability function, } \text{46-7)}
$$

in which $E_F$ is the Fermi energy and $k$ is the Boltzmann constant. (See Eisberg and Resnick, Quantum Physics, second edition, 1985, Wiley, Section 11-4.)

Note that as $T \to 0$, the exponent $(E - E_F)/kT$ in Eq. 46-7 approaches $-\infty$ if $E < E_F$ and $+\infty$ if $E > E_F$. In the first case we have $\rho(E) = 1$ and in the second $\rho(E) = 0$. Thus, at $T = 0$, Eq. 46-7 correctly yields the rectangular form shown in Fig. 46-6a. Equation 46-7 also shows us that the important quantity is not the energy $E$ but rather $E - E_F$, the energy interval between $E$ and the Fermi energy.

Figure 46-7b shows the probability function for $T = 1000$ K, calculated from Eq. 46-7. Note how little it differs from the rectangular form of Fig. 46-6b. Figure 46-7c, found by multiplying Figs. 46-7a and 46-7b, shows the density of occupied states for $T = 1000$ K. Note how little that differs from Fig. 46-6c, the distribution at $T = 0$.

**SAMPLE PROBLEM 46-3**

A cube of copper is 1.00 cm on an edge. How many quantum states lie in the energy interval between $E = 5.00$ eV and $E = 5.01$ eV?

**SOLUTION** These energy limits are so close together that we can say that the answer is

$$
N = n(E) \Delta E V,
$$

where $E = 5.00$ eV, $\Delta E = 0.01$ eV, and $V$ is the volume of the cube. From Eq. 46-3 we have.

$$
n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}
$$

$$
= \left(\frac{8\sqrt{2}\pi}{h^3}\right) (9.11 \times 10^{-31} \text{ kg})^{3/2}
$$

$$
\times \frac{(5.00 \text{ eV})^{1/2}(1.60 \times 10^{-19} \text{ eV})^{1/2}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^{3/2}}
$$

$$
= 9.48 \times 10^{46} \text{ m}^{-3} \text{ eV}^{-1}.
$$

From Eq. 46-8 we have, putting $V = a^3$, where $a$ is the cube edge.

$$
N = n(E) \Delta E a^3
$$

$$
= (1.52 \times 10^{28} \text{ m}^{-3} \text{ eV}^{-1})(0.01 \text{ eV})(1 \times 10^{-2} \text{ m})^3
$$

$$
= 1.52 \times 10^{20}.
$$

(Answer)

**SAMPLE PROBLEM 46-4**

a. What is the probability that a state whose energy is 0.10 eV above the Fermi energy will be occupied? Assume a temperature of 800 K.
\( P(E) = 1 \) and in the

0, Eq. 46-7 correctly

that the important

rather \( E - E_F \), the

the Fermi energy.

probability function for

eq 46-7. Note how liter-

form of Fig. 46-6b.

ing Figs. 46-7a and

tated states for \( T = \)

differences from Fig. 46-6c,

\( 10^{-19} \text{J/eV}^{1/2} \)

\( 1.52 \times 10^{28} \text{m}^{-3} \cdot \text{eV}^{-1} \)

\( V = a^3 \), where \( a \) is the

\( 1 \text{eV} \cdot (1 \times 10^{-2} \text{m})^3 \)

\( \frac{1}{2} \)

a state whose energy is

will be occupied? As-

SOLUTION We can find \( P(E) \) from Eq. 46-7. Let us

first calculate the (dimensionless) exponent in that

equation:

\[
\frac{E - E_F}{kT} = \frac{0.10 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(800 \text{ K})} = 1.45.
\]

Inserting this exponent into Eq. 46-7 yields

\[
P = \frac{1}{e^{1.45} + 1} = 0.19 \text{ or } 19\%.
\]

(Answer)

b. What is the probability of occupancy for a state that is 0.10 eV below the Fermi energy?

SOLUTION The exponent in Eq. 46-7 has the same

numerical value as above but is now negative. Thus

from this equation

\[
P = \frac{1}{e^{1.45} + 1} = 0.81 \text{ or } 81\%.
\]

(Answer)

For states whose energies lie below the Fermi energy we

are often more interested in the probability that the state is not occupied. This is, of course, just \( 1 - P \), or 19% in the present case. An unfilled state in an energy range in which most of the states are filled is called a

hole. We shall see later that this is a very useful concept.

c. What is the probability of occupancy for a state whose energy is equal to the Fermi energy?

SOLUTION For \( E = E_F \) the exponent in Eq. 46-7 is zero and that equation becomes

\[
P = \frac{1}{e^0 + 1} = \frac{1}{1 + 1} = 0.50 \text{ or } 50\%.
\]

(Answer)

This result does not depend on the temperature. We can, in fact, define the Fermi energy for a metal to be that energy for which the probability of occupancy at any temperature is 50%.

46-7 SEMICONDUCTORS

As a comparison of Fig. 46-8 with Fig. 46-4 shows, a semiconductor is like an insulator in that its uppermost filled level (at the absolute zero of temperature) lies at the top of a band. A semiconductor differs from an insulator, however, in that the gap between this filled band and the next vacant band above it is much smaller than for an insulator, so that there is a real possibility for electrons to "jump the gap" into this empty band by thermal agitation. For semiconducting materials, the highest filled band is called the valence band because the elec-
**The Density of Charge Carriers, \( n \)**

Copper has many more charge carriers than does silicon, by a factor of about \(10^{13}\). For copper the carriers are the conduction electrons, present in the number of one per atom. At room temperature, to which Table 46-1 refers, charge carriers in silicon arise only because, at thermal equilibrium, thermal agitation has caused a certain (very small) number of electrons to be raised to the conduction band, leaving an equal number of vacant states (holes) in the valence band.

The holes in the valence band of a semiconductor also serve effectively as charge carriers because they permit a certain freedom of movement to the electrons in that band. If an electric field is set up in a semiconductor, the electrons in the valence band, being negatively charged, will drift opposite the direction of \( E \). This causes the holes to drift in the direction of \( E \). That is, the holes behave like particles carrying a charge \( +e \) and, in all that follows, that is exactly how we shall regard them. Conduction by holes is an important fact of life for semiconductors.

If the concept of a migrating hole seems confusing to you, think of a vacant slot in a parking lot that is otherwise filled with cars. If one of these cars moves into the slot, it fills the slot but creates a new vacant slot in the place it just left. This vacancy, in turn, can be filled by another car, and so on. As the cars move around in this way, we can focus attention on the single migrating vacant slot as it wanders over the lot.

**The Resistivity, \( \rho \)**

At room temperature the resistivity of silicon is considerably higher than that of copper, by a factor of about \(10^{11}\). For both elements, the resistivity is determined by Eq. 46-1. The vast difference in resistivity between copper and silicon can be accounted for by the vast difference in \( n \), the density of charge carriers. (The mean collision time \( \tau \) will also be different for copper and for silicon but the effect of this on the resistivity is swamped by the enormous difference in \( n \).)

**The Temperature Coefficient of Resistivity, \( \alpha \)**

This quantity (see Eq. 28-16) is the fractional change in resistivity per unit change in temperature, or

\[
\alpha = \frac{1}{\rho} \frac{dp}{dT}
\]

The resistivity of copper and other metals increases with temperature \((dp/dT > 0)\). This happens because collisions occur more frequently the higher the temperature, thus reducing \( \tau \) in Eq. 46-1. For metals, the density of charge carriers \( n \) in that equation is independent of temperature.

On the other hand, the resistivity of silicon (and other semiconductors) decreases with temperature \((dp/dT < 0)\). This happens because the density of charge carriers \( n \) in Eq. 46-1 increases rapidly with temperature. The decrease in \( \tau \) mentioned before for metals also occurs for semiconductors but its effect on the resistivity is swamped by the very rapid increase in the density of charge carriers.

**FIGURE 46-10**  (a) A two-dimensional representation of a silicon lattice. Each silicon ion (core charge = +4\( e \)) is bonded to each of its four nearest neighbors by a shared two-electron bond. The red dots show these valence electrons. (b) A phosphorus atom (valence = 5) is substituted for the central silicon atom, creating a donor site. (c) An aluminum atom (valence = 3) is substituted for the central silicon atom, creating an acceptor site.
replacement atoms (it seems pejorative to call them impurities) into the semiconductor lattice, a process called doping. Essentially all practical semiconducting devices today are based on doped material. They are of two varieties, called n-type and p-type; we discuss each in turn.

**n-Type Semiconductors**

Figure 46-10a is a "flattened out" representation of a lattice of pure silicon; compare Fig. 46-1a. Each silicon atom forms a two-electron covalent bond with each of its four nearest neighbors, the electrons involved in the bonding making up the valence band of the sample. In Fig. 46-10b one of the silicon atoms (valence = 4) has been replaced by an atom of phosphorus (valence = 5). As Fig. 46-10b suggests, the "extra" electron is loosely bound to the phosphorus ion core because it is not involved in covalent bonds to neighboring ions. It is far easier for this electron to be thermally excited into the conduction band than it is for one of the silicon valence electrons to be so excited.

The phosphorus atom is called a donor atom because it so readily donates an electron to the conduction band. The "extra" electron in Fig. 46-10b can be said to lie in a localized donor level, as Fig. 46-11a shows. This level is separated from the bottom of the conduction band by an energy gap $E_d$, where $E_d \ll E_g$. By controlling the concentration of donor atoms it is possible to increase greatly the density of electrons in the conduction band.

Semiconductors doped with donor atoms are called n-type semiconductors, the "n" standing for "negative" because the negative charge carriers greatly outnumber the positive charge carriers. The former, called the majority carriers, are the electrons in the conduction band. The latter, called the minority carriers, are the holes in the valence band.

**p-Type Semiconductors**

Figure 46-10c shows a silicon lattice in which a silicon atom (valence = 4) has been replaced by an aluminum atom (valence = 3). Now there is a "missing" electron and it is easy for the aluminum ion to "steal" a valence electron from a nearby silicon atom, thus creating a hole in the valence band.

The aluminum atom is called an acceptor atom because it so readily accepts an electron from the valence band. The electron so accepted moves into a localized acceptor level, as Fig. 46-11b shows. This level is separated from the top of the valence band by an energy gap $E_a \ll E_g$. By controlling the concentration of acceptor atoms it is possible to greatly increase the number of holes in the valence band.

Semiconductors doped with acceptor atoms are called p-type semiconductors, the "p" standing for "positive" because the positive charge carriers in this case greatly outnumber the negative carriers. In p-type semiconductors the majority carriers are the holes in the valence band and the minority carriers are the electrons in the conduction band.

Table 46-2 summarizes the properties of a typical n-type and a typical p-type semiconductor. Note particularly that the donor and acceptor ion cores, although they are charged, are not charge carriers because, at normal temperatures, they remain fixed in their lattice sites.

### Table 46-2

**PROPERTIES OF TWO DOPED SEMICONDUCTORS**

<table>
<thead>
<tr>
<th>Matrix material</th>
<th>Silicon</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dopant</td>
<td>Phosphorus</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Type of dopant</td>
<td>Donor</td>
<td>Acceptor</td>
</tr>
<tr>
<td>Type of semiconductor</td>
<td>n-Type</td>
<td>p-Type</td>
</tr>
<tr>
<td>Dopant valence</td>
<td>$5 (-4 + 1)$</td>
<td>$3 (-4 - 1)$</td>
</tr>
<tr>
<td>Dopant energy gap</td>
<td>45 meV</td>
<td>57 meV</td>
</tr>
<tr>
<td>Majority carriers</td>
<td>Electrons</td>
<td>Holes</td>
</tr>
<tr>
<td>Minority carriers</td>
<td>Holes</td>
<td>Electrons</td>
</tr>
<tr>
<td>Dopant ion core charge</td>
<td>$+e$</td>
<td>$-e$</td>
</tr>
</tbody>
</table>
SAMPLE PROBLEM 46-5

The number density of conduction electrons in pure silicon at room temperature is about $10^{16} \text{ m}^{-3}$. Assume that, by doping the lattice with phosphorus, you want to increase this number by a factor of a million ($10^6$). What fraction of the silicon atoms must you replace with phosphorus atoms? (Assume that, at room temperature, the thermal agitation is effective enough so that essentially every phosphorus atom donates its "extra" electron to the conduction band.)

**Solution**

The density of the phosphorus atoms must be about $(10^{16} \text{ m}^{-3})(10^6)$, or about $10^{22} \text{ m}^{-3}$. The density of silicon atoms in a pure silicon lattice may be found from

$$n_0 = \frac{N_A d}{A},$$

in which $N_A$ is the Avogadro constant, $d$ is the density of silicon, and $A$ is the molar mass of silicon; from Appendix D we find that $d = 2330 \text{ kg/m}^3$ and $A = 28.1 \text{ g/mol}$. Substituting yields

$$n_0 = \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(2330 \text{ kg/m}^3)}{0.0281 \text{ kg/mol}} = 5 \times 10^{28} \text{ m}^{-3}.$$  

The ratio of these two number densities is the quantity we are looking for. Thus

$$\frac{n_0}{n_p} = \frac{5 \times 10^{28} \text{ m}^{-3}}{10^{22} \text{ m}^{-3}} = 5 \times 10^6.$$  

(Answer)

We see that if only one silicon atom in five million is replaced by a phosphorus atom, the number of electrons in the conduction band will be increased by a factor of $10^6$.

How can such a tiny admixture of phosphorus atoms have such a big effect? The answer is that, for pure silicon at room temperature, there were not many conduction electrons to start with. The density of conduction electrons was $10^{16} \text{ m}^{-3}$ before doping and $10^{28} \text{ m}^{-3}$ after doping. For copper, however, the conduction electron density (see Table 46-1) is about $10^{29} \text{ m}^{-3}$. Thus, even after doping, the conduction electron density of silicon remains much less than that of a typical metal such as copper.

46-9 THE p-n JUNCTION

Pass a hypothetical plane across a rod of pure silicon. Dope the rod on one side of the plane with donor atoms (thus creating n-type material) and on the other side with acceptor atoms (thus creating p-type material). You have just made a p-n junction; it is at the heart of essentially all semiconducting devices. Figure 46-12a represents a p-n junction at the imagined moment of its creation. Let us first discuss the motions of the majority carriers, which are electrons in the n-type material and holes in the p-type material.

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*pIn practice, to make a p-n junction one usually starts with, say, n-type material and then diffuses acceptor atoms into the solid sample at high temperature, overcompensating the donor atoms to a certain (controllable) depth below the surface.*
**Motions of the Majority Carriers**

Electrons close to the junction plane will tend to diffuse across it (from right to left in Fig. 46-12), for much the same reason that gas molecules will diffuse through a permeable membrane into a vacuum beyond it. In the same way, holes will tend to diffuse across the junction plane from left to right. Both motions contribute to a diffusion current \(i_{\text{diff}}\), directed from left to right as in Fig. 46-12d.

Recall that \(n\)-type material is studded throughout with donor ions, fixed firmly in their lattice sites. Normally, the positive charges of these ions are compensated electrically by the majority carriers, which are electrons. When an electron diffuses from the \(n\)-type material and through the junction plane, however, it "uncovers" one of these donor ions, thus introducing a fixed positive charge in the \(n\)-type material. When this diffusing electron arrives on the other side of the barrier, it quickly finds a hole and combines with it,\(^a\) thus neutralizing one of the positively charged acceptor ions that are sprinkled throughout the \(p\)-type material, resulting in a fixed negative charge in the \(p\)-type material.

Convince yourself that a hole diffusing through the barrier from left to right has exactly the same end result. Thus a region of fixed positive charge builds up on one side of the barrier and a region of fixed negative charge builds up on the other (Fig. 46-12b). These two regions form the so-called depletion zone; the fixed charges are said to be positive and negative space charge.

The space charge causes a contact potential difference to build up across the junction, as Fig. 46-12c shows. This potential difference is such that it serves as a barrier to limit further diffusion of both electrons and holes across the junction plane. An electron at the junction plane, for example, would be repelled back to its \(n\)-type home by the negative space charge in the \(p\)-type material that faces it across the plane. To complete the picture, let us turn our attention to the minority carriers.

**Motions of the Minority Carriers**

As Fig. 46-11a and Table 46-2 show, although the majority carriers in \(n\)-type material are electrons, there are nevertheless also a few holes, the minority carriers. Likewise in \(p\)-type material, although the majority carriers are holes, there are also a few conduction electrons.

Although the potential difference in Fig. 46-12c acts to retard the motions of the majority carriers—being a barrier for them—it is a downhill trip for the minority carriers, be they electrons or holes. When, by thermal agitation, an electron close to the junction plane is raised from the valence band to the conduction band of the \(p\)-type material in Fig. 46-12a, the contact potential difference causes it to drift steadily from left to right across the junction plane. Similarly, if a hole is created in the \(n\)-type material, it too drifts across to the other side. The space-charge region shown in Fig. 46-12b is effectively swept free of charge carriers by this process and, for that reason, we call it the depletion zone. The current represented by the motions of the minority carriers, called the drift current \(i_{\text{drift}}\), is in the opposite direction to the diffusion current and just compensates it at equilibrium, as Fig. 46-12d shows.

Thus, at equilibrium, a \(p-n\) junction resting on a shelf develops a contact potential difference \(V_0\) between its ends. The diffusion current \(i_{\text{diff}}\) that moves through the junction plane from the direction \(p\) to \(n\) is just balanced by a drift current \(i_{\text{drift}}\) that moves in the opposite direction.

---

\(^a\)An "electron combines with a hole" when the electron drops from the conduction band to the valence band, filling a vacancy in that band.
rowhead is (sufficiently) positive with respect to the other terminal.

Figure 46-15 shows details of the two connections. In Fig. 46-15a—the back-bias arrangement—the battery emf simply adds to the contact potential difference, thus increasing the height of the barrier that the majority carriers must surmount. Fewer of them can do so and, as a result, the diffusion current decreases markedly.

The drift current, however, senses no barrier and thus is independent of the magnitude or direction of the external potential. The nice current balance that existed at zero bias (see Fig. 46-12d) is thus upset and, as shown in Fig. 46-15a, a very small net back-current $i_B$ appears in the circuit.

Another effect of back-bias is to widen the depletion zone, as a comparison of Figs. 46-12b and 46-15a shows. Because the depletion zone contains very few charge carriers, it is a region of high resistivity. Thus its substantially increased width means a substantially increased resistance, consistent with the small value of the back-bias current.

Figure 46-15b shows the forward-bias connection, the positive terminal of the battery being connected to the $p$-type end of the $p$-$n$ junction. Here the applied emf subtracts from the contact potential difference, the diffusion current rises substantially,
and a relatively large net forward current $i_F$ results. The depletion zone narrows, its low resistance being consistent with the large current $i_F$.

46-11 THE LIGHT-EMITTING DIODE (LED)

We are all familiar with the brightly colored numbers that flash at us from cash registers and gasoline pumps. In nearly all cases this light is emitted from an assembly of $p$-$n$ junctions operating as light-emitting diodes (LEDs).

Figure 46-16a shows the familiar seven-segment display from which the numbers are formed. Figure 46-16b shows that each element of this display is the end of a flat plastic lens, at the other end of which is a small LED, possibly about $1 \text{ mm}^2$ in area. Figure 46-16c shows a typical circuit, in which the LED is forward-biased.

How can a $p$-$n$ junction emit light? When an electron at the bottom of the conduction band of a semiconductor falls into a hole at the top of the valence band, an energy $E_g$ is released, where $E_g$ is the gap width. What happens to this energy? There are at least two possibilities. It might be transformed into thermal energy of the vibrating lattice and, with high probability, that is exactly what happens in a silicon-based semiconductor.

In some semiconducting materials, however, conditions are such that the emitted energy can also appear as electromagnetic radiation, the wavelength being given by

$$\lambda = \frac{c}{f} = \frac{c}{E_g/h} = \frac{hc}{E_g}. \quad (46-9)$$

Commercial LEDs designed for the visible region are usually based on a semiconducting material that is a suitably chosen gallium–arsenic–phosphorus compound. By adjusting the ratio of phosphorus to arsenic, the gap width—and thus the wavelength of the emitted light—can be tailored to suit the need.

A question arises: If light is emitted when an electron falls from the conduction band to the valence band, will not light of that same wavelength be absorbed when an electron moves in the other direction, that is, from the valence band to the conduction band? It will indeed. To avoid having all the emitted photons absorbed, it is necessary to have a great surplus of both electrons and holes present in the material, in much greater numbers than would be generated by thermal agitation in the intrinsic semiconducting material.* These are precisely the conditions that result when majority carriers—be they electrons or holes—are injected across the central plane of a $p$-$n$ junction by the action of an external potential difference. That is why a simple intrinsic semiconductor will not serve as an LED. You need a $p$-$n$ junction! To provide lots of majority carriers (and thus lots of photons), the junction should be heavily doped and strongly forward-biased.

LEDs operating in the infrared are much used in optical communication systems based on optical fibers. The infrared region is chosen because the ab-

*If the surplus of electrons and holes is great enough, there may be a population inversion so that conditions for laser action are set up.
SAMPLE PROBLEM 46-6

An LED is constructed from a p-n junction based on a certain Ga-As-P semiconducting material, whose energy gap is 1.9 eV. What is the wavelength of its emitted light?

SOLUTION: From Eq. 46-9 we have

\[
\lambda = \frac{hc}{E_g} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (3.00 \times 10^8 \text{ m/s})}{(1.9 \text{ eV}) (1.60 \times 10^{-19} \text{ J/eV})}
\]

\[
= 6.5 \times 10^{-7} \text{ m} = 650 \text{ nm.}
\]

(Answer)

Light of this wavelength is red.

46-12 THE TRANSISTOR (OPTIONAL)

The devices we have discussed so far have been diodes, that is, two-terminal devices. Here we introduce a three-terminal device, a transistor. As Fig. 46-17 suggests, the function of a transistor is to control current flowing through the device from terminal D (the drain) to terminal S (the source) by applying a potential to terminal G (the gate).

In many applications, particularly those involving optical devices, we only need to be able to turn the drain-source current ON (gate open) or OFF (gate closed). One of these conditions corresponds to a "0" and the other to a "1" in the binary arithmetic on which the computer logic is based. We wish further that the gate terminal draw essentially no current from the circuit to which it is attached, thus interfering with its operation as little as possible. In more formal language, we say that we wish the transistor to have a high input impedance.

The central question proves to be: "How can we control the current in a conductor without making direct electric contact with it?" The perhaps surprising answer is that, by using the variable gate potential that is at our disposal, we can change the effective cross-sectional area of the conductor, going even so far as to reduce it to zero.

Figure 46-15 gives a clue. There we see that, by changing the bias potential of the p-n junction, we can control the width of the depletion zone. Simply by changing a potential we can effectively transform a conductor (n-type or p-type material) into a non-conductor (the depletion zone material). We can use the same trick in the transistor, but with a different geometrical arrangement.

Of the several types of transistor that are in common use, we choose to describe the MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor). Figure 46-19 shows its essential features.

A lightly doped p-type substrate has been etched into two "islands" of heavily doped n-type material, forming the drain D and the source S. These terminals are connected by a thin channel of n-type material, called the n-channel. An insulating layer of silicon dioxide (hence Oxide in the acronym) is deposited on top of the n-channel, providing an electrically isolating barrier between the source and drain. The oxide layer is perforated by a pair of metallic contacts at the source S and drain D. By applying an electric potential to the gate G, we can control the width of the oxide-filled channel between the source and drain.

Deposition of an insulating layer of silicon dioxide and metal leads to a transistor. In the OFF state of the transistor, the gate voltage is small and the oxide layer is thick. In the ON state, the gate voltage is large and the oxide layer is thin. By changing the gate voltage, we can control the width of the oxide-filled channel, which in turn controls the flow of current between the source and drain.
of a transistor, showing the device from the drain side of the current is considered from the drain terminal D, the gate terminal G, or OFF (gate open) or ON (gate closed) or ON (gate closed).

We wish further to draw essentially no current when the transistor is off, thus little as possible. In that we wish the transistor to be: 

"How can we control the drain current without making the transistor conduct?"

The perhaps surprising fact is that we can change the effective resistance of the transistor, going from a high resistance when the transistor is off to a low resistance when the transistor is on.

There we see that, by making the p-n junction, we control the current. Simply by adding the p-n junction, we have effectively transformed a conductor into a non-conductor. We can have a conductor, but with a different kind of property.

The Field-Effect Transistor

Field-Effect Transistor is a transistor that in common parlance is called the MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor).

The substrate has imbedded doped n-type material, source S. These terminal contacts are made of metal and insulated by a thin insulating layer of silicon dioxide (SiO2). Thus a MOSFET has the desired high input impedance, perhaps as high as 10^15 ohms.

Consider first the situation with the source grounded, the drain open, and a positive potential between the source and the drain. The potential difference across the boundary between the n-channel and the p-type substrate will vary from zero at the source end of the channel to V_DS at the drain end. The polarity is such (compare Fig. 46-15a) that the p-n junction that exists at this boundary is back-biased for essentially its full length. A depletion zone will exist at this boundary, increasing in thickness from the source end of the channel to the drain end. For these conditions, the n-channel will not have the same cross-sectional area along its length, being invaded by the depletion zone to a greater and greater extent as one proceeds along the channel from the source toward the drain.

The thickness of the depletion zone along its length can be influenced by the potential that we choose to apply to the gate. If we make the gate negative with respect to the source, electrons will be repelled from the n-channel into the substrate, thus widening the depletion zone, constraining the channel and decreasing the drain-source current. Alternatively, a positive gate potential will attract electrons into the n-channel, narrowing the depletion zone, widening the conducting channel, and increasing the drain-source current. In this way a small change in the gate potential can generate a substantial change in the drain-source current, much as a valve controls the flow of water through a pipe.

Figure 46-20 shows a MOSFET (note the descriptive symbol) connected into a circuit as an amplifier. The input signal is applied to the gate and the output appears as a varying potential difference across a load resistor.
est occupied level coincides with the top of a band; electrons are not able to accept additional kinetic energy from an applied field and so cannot conduct electricity (see Fig. 46-4). In metals the highest occupied level (the Fermi energy) falls somewhere in the middle of a band (Fig. 46-5 and Sample Problems 46-1 and 46-2). The highest occupied level in a semiconductor coincides, at $T = 0$, with the top of a band (the valence band); between it and the next highest band (the conduction band) is small enough so that charge-carrying electrons can "jump" into the conduction band because of thermal agitation (Fig. 46-8). The higher resistivity and the decrease of resistivity with increasing temperature are both easily explained by this model.

**Density of States**
Assuming uniform potential, the density of energy states available to electrons in the partially filled band of Fig. 46-5 is

$$n(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^2} E^{1/2} \quad \text{(density of states)} \quad (46-3)$$

**The Fermi Energy**
At $T = 0$, electrons fill all the states up to the Fermi energy, with states above the Fermi energy being vacant. The Fermi energy corresponding to Eq. 46-3 is

$$E_{F} = \frac{0.121h^2}{m} n^{2/3} \quad \text{(Fermi energy)} \quad (46-6)$$

in which $n$ is the number of conduction electrons per unit volume; see Fig. 46-6.

**The Probability Function**
At temperatures above absolute zero the distribution of occupied states is found by multiplying the density of states by the probability function

$$p(E) = \frac{1}{e^{(E - E_{F})/kT} + 1} \quad \text{(probability function)} \quad (46-7)$$

as illustrated in Fig. 46-7.

**Doping: Donors and Acceptors**
In practice, semiconductors are doped with controlled levels of selected impurities. Donor impurities contribute electrons to the conduction band and produce $n$-type semiconductors. Acceptor impurities contribute holes to the valence band and produce $p$-type semiconductors.

**The $p$-$n$ Junction**
A $p$-$n$ junction (see Fig. 46-12) can serve as a diode rectifier. For forward-biasing ($p$ positive with respect to $n$), the potential barrier is low, the junction is thin, and the forward current is large. For back-biasing, the potential barrier is high, the junction is thick, and the back current is small, usually negligibly so. The junction region itself, regardless of the applied potential difference, is called a depletion layer. It is virtually free of charge carriers and behaves like a somewhat leaky insulating slab.

**The Light-Emitting Diode (LED)**
A $p$-$n$ junction can, under certain circumstances, convert the energy lost by a charge carrier crossing the barrier into visible light whose wavelength is

$$\lambda = \frac{h\alpha}{E_{g}} \quad (46-9)$$

$E_{g}$ being the energy gap width.

**Transistors**
A transistor is a three-terminal solid-state device in which the current flowing from the drain to the source is controlled by varying the potential of a gate; see Fig. 46-18. Figure 46-19 shows a MOSFET-type transistor, in which a small variation of the potential difference $V_{gs}$ between the gate $G$ and the source $S$ has a major controlling effect on the current $i_{ds}$ between the drain $D$ and the source $S$.
28. A semiconductor contains equal numbers of donor and acceptor impurities. Do they cancel each other in their electrical effects? If so, what is the mechanism? If not, why not?

29. Germanium and silicon are similar semiconducting materials whose principal distinction is that the gap width \( E_g \) (see Fig. 46-8) is 0.67 eV for the former and 1.14 eV for the latter. If you wished to construct a \( p-n \) junction in which the back current is to be kept as small as possible, which material would you choose and why?

30. Consider two possible techniques for fabricating a \( p-n \) junction. (a) Prepare separately an \( n \)-type and a \( p \)-type sample and join them together, making sure that their abutting surfaces are plane and highly polished. (b) Prepare a single \( n \)-type sample and diffuse an excess acceptor impurity into it from one face, at high temperature. Which method is preferable and why?

31. In a \( p-n \) junction we have seen that electrons and holes may diffuse, in opposite directions, through the junction region. What is the eventual fate of each such particle as it diffuses into the material on the opposite side of the junction?

32. Does the diode rectifier whose characteristics are shown in Fig. 46-13 obey Ohm's law? What is your criterion for deciding?

33. We have seen that a simple intrinsic (undoped) semiconductor cannot be used as a light-emitting diode. Why not? Would a heavily doped \( n \)-type or \( p \)-type semiconductor work?

34. Explain how the MOSFET device of Fig. 46-19 works.

35. The acronym MOSFET stands for Metal-Oxide-Semiconductor Field-Effect Transistor. What is the significance of each of these terms as applied to the device shown in Fig. 46-19?

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**EXERCISES & PROBLEMS**

**SECTION 46-5 METALS: QUALITATIVE**

1E. At what pressure, in atmospheres, would an ideal gas have a density of molecules equal to the density of the conduction electrons in copper \( (= 8.43 \times 10^{28} \text{ m}^{-3}) \)? Assume \( T = 300 \text{ K} \).

2E. Gold is a monovalent metal with a molar mass of 197 g/mol and a density of 19.3 g/cm³. Calculate the density of charge carriers.

3P. Calculate the number of particles per cubic meter for (a) the molecules of oxygen gas at 0°C and 1.0-atm pressure and (b) the conduction electrons in copper. (c) What is the ratio of these numbers? (d) What is the average distance between particles in each case? Assume that this distance is the edge of a cube whose volume is equal to the volume per particle. (See Sample Problem 28-3.)

4E. The density and molar mass of sodium are 971 kg/m³ and 23 g/mol, respectively; the radius of the ion \( \text{Na}^+ \) is 98 pm. (a) What fraction of the volume of metallic sodium is available to its conduction electrons? (b) Carry out the same calculation for copper. Its density, molar mass, and ionic radius are, respectively, 8960 kg/m³, 63.5 g/mol, and 135 pm. (c) For which of these two metals do you think the conduction electrons behave more like a free electron gas?

**SECTION 46-6 METALS: QUANTITATIVE**

5E. Use Eq. 46-6 to verify that the Fermi energy of copper is 7.0 eV. (Note, from Sample Problem 46-1, that the density of charge carriers in copper is \( 8.43 \times 10^{28} \text{ m}^{-3} \).)

6E. (a) Show that Eq. 46-3 can be written as

\[ n(E) = C E^{-2/3}, \]

where \( C = 6.78 \times 10^{37} \text{ m}^{-3} \cdot \text{eV}^{-1/3} \). (b) Use this relation to verify a calculation of Sample Problem 46-8, namely, that for \( E = 5.00 \text{ eV}, n(E) = 1.52 \times 10^{10} \text{ m}^{-3} \cdot \text{eV}^{-1} \).

7E. Calculate the density \( n(E) \) of conduction electron states in a metal for \( E = 8.0 \text{ eV} \) and show that your result is consistent with the curve of Fig. 46-6a.

8E. What is the probability that a state 0.062 eV above the Fermi energy is occupied at (a) \( T = 0 \text{ K} \) and (b) \( T = 300 \text{ K} \)?

9E. The Fermi energy of copper is 7.0 eV. For copper at 1000 K, (a) find the energy at which the occupancy probability is 0.90. For this energy, evaluate (b) the density of states and (c) the density of occupied states.

10E. Show that Eq. 46-6 can be written as

\[ E_F = A n^{2/3}, \]

where the constant \( A \) has the value \( 3.65 \times 10^{-15} \text{ m}^2 \cdot \text{eV}^{-1/3} \).

11E. The density of gold is 19.3 g/cm³. Each atom contributes one conduction electron. Calculate the Fermi energy of gold.

12E. Figure 46-7c shows the density-of-occupied states \( n_o(E) \) of the conduction electrons in a metal at 1000 K. Calculate \( n_o(E) \) for copper for the energies \( E = 4.00, 6.75, 7.00, 7.25, \) and 9.00 eV. The Fermi energy of copper is 7.00 eV.

13E. It can be shown that the conduction electrons in a metal behave like an ideal gas of the ordinary kind if the temperature is high enough. In particular, the temperature must be such that \( kT \gg E_F \), the Fermi energy. What temperatures are required for copper \( (E_F = 7.0 \text{ eV}) \) for this to be true? Study Fig. 46-7c in this connection and note that we have wished to refer to this here.

14E. The Fermi level in a metal is determined by the density of states and the acceptor and donor impurities. (a) What are the energies at which the Fermi level is occupied? (b) At what temperatures is the Fermi level 5.0 eV above the conduction band in copper?

15E. The Fermi level in Si is 2.70 eV above the conduction band and 4.3 eV below the valence band. Determine the Fermi energy for copper.

16E. Show that copper has a density of free electrons equal to the density of metallic copper.

17P. Show that for \( n(E) \) in a metal, the Fermi energy is the energy at which the density of states is maximum.

18P. Zinc is a metal consisting of atoms of atomic number 30. (a) The Fermi energy of zinc is 5.6 eV. What is this in terms of the states per \( \text{cm}^3 \) in the \( n \)-band of zinc? (b) For needed data, consult Table C-3.

19P. Silver is a metal of density 10.5 g/cm³. Each atom contributes one conduction electron. Calculate the Fermi energy of silver.

20P. Calculate the density of states per \( \text{cm}^3 \) in the conduction band of zinc at 0 K. (Hint: The density of states per \( \text{cm}^3 \) is \( 10.3 \times 10^{29} \text{ states/} \text{cm}^3 \) for copper at 0 K.)

21P. A metal has a density of 9 g/cm³. Each atom contributes one conduction electron. Calculate the Fermi energy of the metal.

22P. Show that the Fermi energy in copper is 7.0 eV.

23P. (Sample Problem 46-8) The carrier density of copper is \( 2 \times 10^{10} \text{ cm}^{-3} \) for \( 300 \text{ K} \). What is the average distance between charge carriers at this temperature?

24P. The energy width of the conduction band in copper is 7.0 eV. What is the energy at which the density of states is maximum for the conduction band? (Use the equation for the density of states per \( \text{cm}^3 \) in the conduction band of a metal given in the text.)

25P. Explain why the density of carriers in a metal is given by Eq. 46-6.

(Hint: The density of states in a metal is \( (2\pi)^3 n_o(E) \).)
exercise one that electrons and directions, through the usual fate of each such letal on the opposite side, free characteristics are law? What is your crictisic (undoped) semih bright-emitting diode. Why + or p-type semiconductors of Fig. 46-19 works, poly for Metal-Oxide-Semiconductor. What is the signif— implied to the device.

(14E) The Fermi energy of silver is 5.5 eV. (a) At T = 0°C, what are the probabilities that states at the following energies are occupied: 4.4 eV, 5.4 eV, 5.5 eV, 5.6 eV, 6.4 eV? (b) At what temperature will the probability that a state at 5.6 eV is occupied be 0.16?

15E. The Fermi energy of aluminum is 11.6 eV; its density is 2.70 g/cm³, and its molar mass is 27.0 g/mol (see Appendix D). From these data, determine the number of free electrons per atom.

16P. Show that the occupancy probabilities of two states whose energies are equally spaced above and below the Fermi energy add up to unity.

17P. Show that the probability that a hole exists at a state of energy E is given by

\[ p_h = \frac{1}{e^{(E - E_F)/kT} + 1} \]

(Hint: The existence of a hole means that the state is unoccupied; convince yourself that this implies that \( p_h = 1 - p_e \)).

18P. Zinc is a bivalent metal. Calculate (a) the number of conduction electrons per cubic meter, (b) the Fermi energy \( E_F \), (c) the Fermi speed \( v_F \), and (d) the de Broglie wavelength corresponding to this speed. See Appendix D for needed data from Appendix D.

19P. Silver is a monovalent metal. Calculate (a) the number of conduction electrons per cubic meter, (b) the Fermi energy \( E_F \), (c) the Fermi speed \( v_F \), and (d) the de Broglie wavelength corresponding to this speed. Extract needed data from Appendix D.

20P. White dwarf stars represent a late stage in the evolution of stars like the sun. They become dense enough and hot enough that we can analyze their structure as a solid in which all Z electrons per atom are free. For a white dwarf with a mass equal to that of the sun and a radius equal to that of the Earth, calculate the Fermi energy of the electrons. Assume the atomic structure to be represented by iron atoms, and \( T = 0 \) K.

21P. A neutron star can be analyzed by techniques similar to those used for ordinary metals. In this case the neutrons (rather than electrons) obey the probability function, Eq. 46-7. Consider a neutron star of 2.0 solar masses with a radius of 10 km. Calculate the Fermi energy of the neutrons.

22P. Show that the density-of-states function given by Eq. 46-8 can be written in the form

\[ n(E) = 1.5nF/3/2E/F \]

Explain how it can be that \( n(E) \) is independent of the material when the Fermi energy \( E_F \) (7.0 eV for copper, 9.4 eV for zinc, and so on) appears explicitly in this expression.

23P. Estimate the number \( N \) of conduction electrons in a metal that have energies greater than the Fermi energy as follows. Strictly, \( N \) is given by

\[ N = \int_{E_F}^{E+4kT} n(E) \delta(E) \, dE \]

where \( E_F \) is the energy at the top of the band. By studying Fig. 46-7e, convince yourself that, to a good degree of approximation, this expression can be written as

\[ N = \int_{E_F}^{E+4kT} n(E) \delta(E) \, dE \]

By substituting the density of states function, evaluated at the Fermi energy, show that this yields for the fraction \( f \) of conduction electrons excited to energies greater than the Fermi energy,

\[ f = \frac{N}{n} = \frac{3kT}{E_F} \]

Why not evaluate the first integral above directly without resorting to an approximation?

24P. Use the result of Problem 23 to calculate the fraction of excited electrons in copper at temperatures of (a) absolute zero, (b) 300 K, and (c) 1000 K.

25P. At what temperature will the fraction of excited electrons in lithium equal 0.013? The Fermi energy of lithium is 4.7 eV. See Problem 23.

26P. Silver melts at 961°C. At the melting point, what fraction of the conduction electrons are in states with energies greater than the Fermi energy of 5.5 eV? See Problem 23.

27P. Show that, at the absolute zero of temperature, the average energy \( \bar{E} \) of the conduction electrons in a metal is equal to \( \frac{1}{2}E_F \), where \( E_F \) is the Fermi energy. (Hint: Note that, by definition of average, \( \bar{E} = \frac{1}{n} \int n(E) \bar{E} \, dE \).)

28P. Use the result of Problem 27 to calculate the total translational kinetic energy of the conduction electrons in 1.0 cm³ of copper at absolute zero.

29P. (a) Using the result of Problem 27, estimate how much energy would be released by the conduction electrons in a penny (assumed all copper; mass = 3.1 g) if we could suddenly turn off the Pauli exclusion principle. (b) For how long would this amount of energy light a 100-W lamp? Note that there is no known way to turn off the Pauli principle!

SECTION 46-8 DOPING

30E. The probability function of Section 46-6 can be applied to semiconductors as well as to metals. In semiconductors, \( E \) is the energy above the top of the valence band. The Fermi level for an intrinsic semiconductor is nearly midway between the top of the valence band and the bottom of the conduction band. For germanium these bands are separated by a gap of 0.67 eV. Calculate the probability that (a) a state at the bottom of the conduction band is occupied and (b) a state at the top of the valence band is unoccupied at 300 K.
31E. Pure silicon at room temperature has an electron density in the conduction band of approximately $1 \times 10^{15} \text{ m}^{-3}$ and an equal density of holes in the valence band. Suppose that one of every $10^7$ silicon atoms is replaced by a phosphorus atom. (a) Which type will this doped semiconductor be, $n$ or $p$? (b) What charge carrier density will the phosphorus add? (See Appendix D for needed data on silicon.) (c) What is the ratio of the charge carrier density in the doped silicon to that in the pure silicon?

32E. What mass of phosphorus would be needed to dope a 1.0-g sample of silicon to the extent described in Sample Problem 46-5?

33P. Doping changes the Fermi energy of a semiconductor. Consider silicon, with a gap of 1.11 eV between the valence and conduction bands. At 300 K the Fermi level of the pure material is nearly at the midpoint of the gap. Suppose that it is doped with donor atoms, each of which has a state 0.15 eV below the bottom of the conduction band, and suppose further that doping raises the Fermi level to 0.11 eV below the bottom of that band. (a) For both the pure and doped silicon, calculate the probability that a state at the bottom of the conduction band is occupied. (b) Also calculate the probability that a donor state in the doped material is occupied. See Fig. 46-21.

34P. A silicon sample is doped with atoms having a donor state 0.11 eV below the bottom of the conduction band. (a) If each of these states is occupied with probability $5.00 \times 10^{-3}$ at $T = 300$ K, where is the Fermi level relative to the top of the valence band? (b) What then is the probability that a state at the bottom of the conduction band is occupied? The energy gap in silicon is 1.11 eV.

35P. In a simplified model of an intrinsic semiconductor (no doping), the actual distribution in energy of states is replaced by one in which there are $N_v$ states in the valence band, all of these states having the same energy $E_v$, and $N_c$ states in the conduction band, all of these states having the same energy $E_c$. The number of electrons in the conduction band equals the number of holes in the valence band. (a) Show that this last condition implies that

$$\frac{N_c}{e^{(E_c-E_v)/kT} + 1} = \frac{N_v}{e^{-(E_v-E_c)/kT} + 1}.$$  

![Figure 46-21](Problem 35.)

36E. When a photon enters the depletion region of a $p$-$n$ junction, electron-hole pairs can be created as electrons absorb part of the photon's energy and are excited from the valence band to the conduction band. These junctions are thus often used as detectors for photons, especially for x-rays and nuclear gamma rays. When a 662-keV gamma-ray photon is totally absorbed by a semiconductor with an energy gap of 1.1 eV, on the average how many electron-hole pairs are created?

37P. For an ideal $p$-$n$ junction diode, with a sharp boundary between the two semiconducting materials, the current $i$ is related to the potential difference $V$ across the diode by

$$i = i_s(e^{qV/kT} - 1),$$

where $i_s$, which depends on the materials but not on the current or potential difference, is called the reverse saturation current. $V$ is positive if the junction is forward-biased and negative if it is back-biased. (a) Verify that this expression predicts the behavior expected of a diode by sketching $i$ as a function of $V$ over the range $-0.12 V < V < 0.12 V$. Take $T = 300$ K and $i_s = 5.0 \text{ nA}$. (b) For the same temperature, calculate the ratio of the current for a 0.50-V forward-bias to the current for a 0.50-V back-bias.

SECTION 46-11 THE LIGHT-EMITTING DIODE (LED)

38E. (a) Calculate the maximum wavelength that will produce photoconduction in diamond, which has a band gap of 7.0 eV. (b) In what part of the electromagnetic spectrum does this wavelength lie?

39E. In a particular crystal, the highest occupied band of states is full. The crystal is transparent to light of wavelength longer than 295 nm but opaque at shorter wavelengths. Calculate, in electron-volts, the gap between the highest occupied band and the next (empty) band.

40E. The KCl crystal has a band gap of 7.6 eV above the topmost occupied band, which is full. Is this crystal opaque or transparent to light of wavelength 140 nm?

41P. Fill in the seven-segment display shown in Fig. 46-16 to show how all 10 numbers may be generated. (b) If the segments are displayed randomly, in what fraction of the displays will each of the seven segments be used?
Fermi level is in the gap from both bands and are excited from a band. These junctions with a sharp bounding are created as electrons and photons, especially for a $662$-keV gamma-semiconductor with a wavelength of $7.6$ eV above the full. Is this crystal opaque at a wavelength of $140$ nm? (a) If $5.0$ nA. (b) For the ratio of the current for a $0.50$-V back-bias.

With increased understanding of their optoelectronic properties, LCDs moved toward higher-information-density applications involving pictures, as well as numbers and letters, required, for example, by lap-top PCs (Fig. 2) and color TVs. Liquid crystal color displays are awesome and need much less space than conventional displays. The LCD display package is flat and lightweight, and consumes little electrical power even when backlighting is added, as in the display shown in Fig. 2.

The purpose of this essay is to outline the physics involved in the operation of one picture element, or pixel, in the simplest LCD, the twisted nematic display. While the basic physics...
ics in the operation of one pixel is, roughly speaking, the same as in high-information-density displays, the passage from low-density to high-density displays required considerable R&D effort involving chemists, optoelectronic engineers, physicists, materials scientists, and mathematicians, as well as the innovation of sophisticated manufacturing processes. Because of the expanding role computers play in our daily life, it is expected that before the turn of the century, LCDs will dominate the high-information-density display market.

Molecules That Align

In a crystal, molecules and atoms are organized in a three-dimensional structure where each atom or molecule does not move far from its position in the structure. Liquid phases have no structural order, so, while keeping close to each other, molecules or atoms move freely in the liquid state. Molecules are made up of many atoms and can exhibit a new state of matter called liquid crystal that is not as ordered as a solid crystal but also not as disordered as the usual liquid state, referred to as the isotropic liquid state.

Molecular shape plays an important role in determining liquid crystal phases. A molecule can be spherical, rodlike or cigar-shaped, disklike, bowl-shaped, or some other complex shape. Liquid crystal phases form when rod-shaped molecules spontaneously align while remaining liquid. In the liquid crystal state called nematic, which is used in the displays shown in Figs. 1 and 2, a degree of alignment is maintained despite the rapid, random thermal motion of individual molecules characteristic of the liquid state. The property of certain molecules to align is called long-range orientational order. It is the property that characterizes liquid crystals.

Molecules forming the nematic liquid crystal used in displays are 20 Å long and about 5 Å in diameter. The nematic state is a result of many molecules aligning cooperatively in the same direction. In discussing liquid crystal phases, one thinks in terms of many molecules, not one. Instead of referring to a single molecule, we talk about the collection of molecules in the nematic state and refer to the direction of alignment, \( \mathbf{n} \), in short-hand form, \( n \), where \( n \) is a unit vector oriented in the direction of alignment, \( \mathbf{n} \), that determines the preferred direction for the molecules to align. The liquid crystal phase is determined by small forces such as weak electric fields or surface forces. When the molecules align in the direction of the field, n, they can be said to be in the nematic state.

Important physical principles that determine how LCDs work are as follows:

1. The orientation of \( \mathbf{n} \) can be determined by small forces such as weak electric fields or surface forces.
2. When two different forces compete for the orientation of \( \mathbf{n} \), the response of \( \mathbf{n} \) can be tuned by varying one of the forces.

Determining Alignment

Surface alignment is achieved by using the orientation of a monomolecular layer of liquid crystal adsorbed on a glass surface. The molecules are oriented perpendicular to the surface. The electric field \( \mathbf{E} \) may be applied to a uniaxially aligned nematic liquid crystal between two parallel plates coated with a transparent conducting electrode. The molecules of the liquid crystal prefer to align parallel to the field, \( \mathbf{E} \), and perpendicular to the surface.

FIGURE 3 Structure of the nematic liquid crystal: rodlike molecules align. The alignment is not perfect because molecules in the liquid state are in rapid, random thermal motion.

![Nematic Alignment](image)
The behavior of polarized light traveling parallel to $\mathbf{n}$ is different from that of polarized light traveling perpendicular to $\mathbf{n}$. Determining the Orientation of $\mathbf{n}$

Surface forces provide a useful way to select $\mathbf{n}$. For example, when microscope slides are buffed in a single direction many times on a piece of white filter paper, a nematic liquid crystal in contact with the buffed surface orientes with $\mathbf{n}$ following the buffing direction. Conventional wisdom is that this is magic but rather that oil from the fingers is transferred to the glass surfaces. The buffing process produces microscopic grooves in the oil, providing an easy direction for the alignment of $\mathbf{n}$.

To make a liquid crystal sample with a uniform $\mathbf{n}$, the nematic liquid crystal is sandwiched between two baffled glass plates with the buffed surfaces facing the liquid and the directions of buffing on the two plates parallel to each other. In such a sample, $\mathbf{n}$ is parallel to the glass surfaces and parallel to the buffing direction throughout the whole sample.

Another way to orient $\mathbf{n}$ is with an electric field $\mathbf{E}$. In some materials, $\mathbf{n}$ aligns parallel to $\mathbf{E}$ while in others, $\mathbf{n}$ aligns perpendicular to $\mathbf{E}$. Materials for which $\mathbf{n}$ aligns parallel to $\mathbf{E}$ are called positive materials. The vector $\mathbf{n}$ orients in both ac and dc fields as there is no energy difference between the state $\mathbf{n}$ parallel to $\mathbf{E}$ and the state $\mathbf{n}$ antiparallel to $\mathbf{E}$. Typically, LCDs use ac fields and positive materials.

When surface forces at electrodes pin $\mathbf{n}$ perpendicular to $\mathbf{E}$, then a large-enough voltage has to be applied to a positive material before $\mathbf{n}$ responds: a threshold voltage $V_\text{th} = E_0/d$ (d is the distance between the electrodes) has to be applied for $\mathbf{E}$ to successfully compete with surface forces in the reorientation of $\mathbf{n}$. For $V > V_\text{th}$, the component of $\mathbf{n}$ parallel to $\mathbf{E}$ is determined by the relative strengths of the applied electric field and the pinning force. A value of $V_\text{th} = 2V$ is a typical threshold voltage for nematics. As this is small by industry standards, relatively cheap electronics are needed to drive LCDs.

The Twisted Nematic

This is the configuration widely used in LCDs. A positive nematic liquid crystal is sandwiched between buffed glass plates that have transparent electrodes evaporated onto the buffed sides. The two plates are oriented with their respective buffing directions nearly perpendicular to each other. The director $\mathbf{n}$ twists nearly 90° smoothly between the two plates. A typical distance between electrodes is 6 μm. The polarization direction of incoming polarized light follows the gentle twist in $\mathbf{n}$. When $V < V_\text{th}$, a pixel in the twisted state looks bright when viewed between crossed polarizers. The upper part of Fig. 4 shows this.

When $V > V_\text{th}$, the optic axis is parallel to the electric field, destroying the twisted structure. The polarization direction of incoming polarized light does not rotate as the light traverses the liquid crystal and is extinguished by the second polarizer. When $V = V_\text{th}$, the pixel is as dark as allowed by the crossed polarizers. This is shown in the lower part of Fig. 4; the state used to show characters on a monochrome display. Intermediate shades of grey are observed for intermediate values of $V$. When the electric field is turned off, the orientation of $\mathbf{n}$ is again determined by surface forces and $\mathbf{n}$ relaxes back to the initially twisted configuration.

The relaxation time depends on the distance between the electrodes and on an orientational diffusion constant.

Liquid crystal displays require little power because ambient light is used. Even with additional power consumption from backlighting used in higher-information-density displays, such as Fig. 2, the total power to run LCDs is smaller than conventional displays. Another LCD feature is that it works in white light, and so color displays can be made by adding color filters to the glass substrates. Indeed, liquid crystals make stunning color displays. Typical LCD switching times are about 10 ms. For some applications this is too slow. A major part of LCD research is to find ways to reduce this time.

The Effect of Temperature

Liquid crystals are useful for studying the general problem of how order in matter is created and destroyed. Indeed, some scientists are using nematic liquid crystals to model the birth of the universe immediately after the Big Bang! Still others are using liquid crystal materials to learn about nonlinear, nonequilibrium dynamic processes and transitions to chaos.

When rodlike molecules align, they are more densely packed than when randomly oriented. For example, matches arranged in a box...
are more closely packed than when thrown at random onto a table. At some low range of temperatures, rod-shaped molecules prefer to pack in the denser aligned state. As temperature is increased, the system becomes less dense and more energetic. The chance that all the molecules choose the same alignment is reduced. At a special temperature, called the transition temperature, there is a sudden change to another liquid state—without long-range orientational order. A phase transition takes place from the nematic liquid crystal phase to the isotropic liquid.

This transition is easily observed when a nematic liquid crystal* is heated. In samples a few millimeters thick, the nematic state is translucent, like frosted glass, because of fluctuations in the alignment caused by thermal motion of the molecules. These fluctuations cause local variations in the index of refraction that scatter light. When heated into the isotropic liquid state, a thick sample becomes transparent because the molecules are now uniformly disordered.

Thus the LCD operates in a certain temperature range. If the temperature is too high, the material transforms to the isotropic liquid, losing orientational order, and the display loses contrast. If the temperature is too low, the material transforms to a more ordered liquid crystal state or, perhaps, even the crystalline state, and its orientation cannot be easily changed. Although materials are known that have a nematic state from $-50^\circ$C to $400^\circ$C, the nematic temperature range in any one compound is typically between 1 K and 20–30 K. To obtain the wide temperature range needed for applications, several different compounds are mixed together.

Chiral Liquid Crystals

In a flat panel display such as used in today’s lap-top computers, picture information reaches the screen through rows of electrodes on one substrate and columns of electrodes on the other. The intersection of the electrodes forms a grid of pixels. In a simple matrix drive system, electric signals are applied to the row and column electrodes with the proper timing to select the target pixel. Because a pixel responds to the rms voltage on a line, an iron law prevails that limits the number of lines that can be addressed. In the case of the twisted nematic, this is about 200 lines. When the nematic is twisted even more (between 180° and 270°, now called a super-twisted nematic, STN, display), the electro-optic response becomes steeper, making 768 lines available. But to make an STN display requires a liquid crystal that spontaneously twists: a chiral liquid crystal.

Chiral crystals have a characteristic hand that is a characteristic of the material. Cholesteric liquid crystals are one example of chiral liquid crystal phases. The nematic is the special case of a cholesteric where the pitch is infinite.

The two ways of twisting are identified as left-handed and right-handed. In the schematic shown in Fig. 4, the director twists in a left-handed manner. To see this, extend the thumb of the left hand in the direction of twist, that is, perpendicular to the glass plates with thumb tip toward the second plate. On going through the liquid from the plate called "one" to the plate called "two," fingers of the left hand naturally curl to maintain alignment with the director. Does it matter which plate is called "one"? Fingers of the right hand curl in the opposite sense and follow a right-handed twist. It costs the same energy for a nematic liquid crystal to twist in a left-handed sense as in a right-handed sense. It costs no energy for a cholesteric liquid crystal to twist in one sense and a great deal to twist in the other.

Figure 5 shows half a pitch of the left-handed cholesteric structure. To make clear the three-dimensional property of this helical structure, cylinders, rather than the simple lines used in Fig. 3, are used to depict n in Fig. 5. As cholesterics are also three-dimensional liquids, the cylinders are not on perfect rows or columns. A left hand is also shown in Fig. 5 to illustrate how handedness is determined. In the same way that the mirror image of a right hand is a left hand, the mirror image of a right-handed twisted structure is a left-handed one.

Cholesteric liquid crystals have an interesting way of interacting with light. They are optically active. While it is nontrivial to synthesize optically active materials in the laboratory, nature does it all the time. Indeed, in a Dorothy Sayers detective story, a deadly case of murder by mushroom poisoning was solved when it was discovered that the mushroom poison ingested by the victim was not opti...
characteristic of the cholesteric liquid crystal is the presence of chiral liquid crystals, which are helical. The nematic phase is the special case of cholesteric where the pitch of the helical structure is zero.

Ways of twisting are identified as right-handed and left-handed. The schematic shown in Fig. 3 is used to depict n twist. The director is the cylindrical vector of constant unit length to which all points of the crystal relate. The director at a point always points in the same direction, although the director itself rotates within the plane perpendicular to the twist direction.

A twisted nematic display consists of two polarizing plates with their optic axes parallel to one another. A third plate is placed at an angle to the optic axes of the first two plates. This third plate is called the analyser plate.

Cholesteric liquid crystals are also three-dimensional. They are optically active. It had been synthesized in a university chemistry laboratory by a murderer, not by the murderer, it turned out.

Unpolarized white light can be thought of as composed of both left and right circular polarizations rotating at all the frequencies corresponding to all the wavelengths constituting white light. A cholesteric liquid crystal with a pitch comparable to the wavelength of light transmits all the incident light except circularly polarized light with the opposite hand to its twisted structure and of a particular wavelength corresponding to its pitch and the angle of incidence of the light. When light travels in the direction of twist, the wavelength equal to the pitch is rejected. Light traveling obliquely to the twist direction "sees" a shorter pitch, and so a shorter wavelength is scattered back. Viewed in ordinary diffuse daylight, the scattering of different wavelengths in different directions produces a striking display of vivid colors "resembling the appearance of a peacock's feathers."

In some materials the pitch depends sensitively on temperature. These materials are used as thermometers to translate small temperature differences into different colors. In some countries, newborns in a nursery have a small adhesive patch of encapsulated temperature-sensitive cholesteric on their foreheads. A single nurse can identify at a glance when one of them has a fever. A temperature map of complicated surfaces — for example, parts of the human body — is obtained by spraying a thin cholesteric liquid crystal layer onto the surface in question. In the medical field, temperature-sensitive cholesteric are an inexpensive, noninvasive way to detect at an early stage tumors, cancerous growth, and, in some Third World countries, leprosy, as the temperature of these tissues is usually different from the temperature of healthy tissue.

Discussion Topics

1. How would the two states of the LCD in Fig. 4 look if the polarizers were parallel to each other?
2. An idealized LCD is shown in Fig. 4 with the director exactly parallel to the glass plates. This is not practical for a display because there are two ways for n to turn toward E. Why is this bad for a display? Draw the picture with n rotated by a small angle θ toward E in one region and −θ in another. How do the two regions connect? This problem is overcome by treating the glass substrates so that n is inclined a little to them. Why does this work? How do you think a small angle affects the sharpness of the optical response above threshold?

3. Harder question. A little bit of cholesteric material is added to most twisted nematic displays to ensure that only one hand is preferred in the twisted structure. Why is this a good idea? Draw a picture of a left-handed twist in one part of the sample and a right-handed one in the other. How do they connect up?

4. While V does not depend on the distance between electrodes, how does the time to turn off a pixel depend on this distance? Hint: The dimensions of a diffusion constant, D0, are square centimeters per second and the distance between electrodes, d, is in centimeters. Put D0 and d together to get a quantity that has units of time.

References for Further Reading

To learn more about:


