II. The Hall Effect

Evolution of Resistance Concepts

The Hall Effect and the Lorentz Force

The van der Pauw Technique

Evolution of Resistance Concepts

Electrical characterization of materials evolved in three levels of understanding. In the early 1800's the resistance $R$ and conductance $G$ were treated as measurable physical quantities obtainable from two-terminal I-V measurements (i.e., current $I$, voltage $V$). Later, it became obvious that the resistance alone was not comprehensive enough, since different shapes of sample gave different resistance values. This led to the understanding (second level) that an intrinsic material property like resistivity (or conductivity) is required that is not influenced by the particular geometry of the sample. For the first time, this allowed scientists to quantify the current-carrying capability of the material, and carry out meaningful comparisons between different samples.

By the early 1900's, it was realized that resistivity was not a fundamental material parameter, since different materials can have the same resistivity. Also, a given material might exhibit different values of resistivity, depending upon how it was synthesized. This is especially true for semiconductors, where resistivity alone could not explain all observations. Theories of electrical conduction were constructed with varying degrees of success, but until the advent of quantum mechanics, no generally acceptable solution to the problem of electrical transport was developed. This led to the definitions of carrier density $n$ and mobility $\mu$ (third level of understanding) which are capable of dealing with even the most complex electrical measurements today.

The Hall Effect and the Lorentz Force

The basic physical principle underlying the Hall effect is the Lorentz force. When an electron moves along a direction perpendicular to an applied magnetic field, it experiences a force acting normal to both directions, and moves in response to this force and the force effected by the internal electric field. For an n-type bar-shaped semiconductor shown in Fig. 1, the free carriers are predominately electrons of bulk density $n$. We assume that a constant current $I$ flows along the x-axis from left to right in the presence of a z-directed magnetic field. Electrons subject to the Lorentz force initially drift away from the current flow line toward the negative y-axis direction, thus resulting in an excess surface electrical charge on the side of the sample. This excess surface charge results in the Hall voltage, a potential drop across the two sides of the sample. (Note that the force on holes is toward the same side because of their opposite velocity and positive charge.)

This transverse voltage is the Hall voltage $V_H$ and its magnitude is equal to $IB/qnd$, where $I$ is the applied current, $B$ is the applied magnetic field intensity, $d$ is the sample thickness, and $q (1.602 \times 10^{-19} C)$ is the...
elementary charge. In some cases, it is convenient to use layer or sheet density \( (n_s = nd) \) instead of bulk density. One then obtains the equation

\[
n_s = IB/q|V_H|.
\]

Thus, by measuring the Hall voltage \( V_H \) and from the known values of \( I, B, \) and \( q \), one can determine the sheet density \( n_s \) of charge carriers in semiconductors. If the measurement apparatus is set up as described later in Section III, the Hall voltage is negative for n-type semiconductors (and positive for p-type semiconductors).

The sheet resistance \( R_S \) of the semiconductor can be conveniently determined by use of the van der Pauw resistivity measurement technique. Since sheet resistance involves both sheet density and mobility, one can determine the Hall mobility from the equation

\[
\mu = |V_H|/R_SI_B = 1/(qn_sR_S).
\]

If the conducting layer thickness \( d \) is known, one can determine the bulk resistivity \( (\rho = R_Sd) \) and the bulk density \( (n = n_s/d) \).

**The van der Pauw Technique**

In order to determine both the mobility \( \mu \) and the sheet density \( n_s \), a combination of a resistivity measurement and a Hall measurement is needed. We discuss here the van der Pauw technique which, due to its convenience, is widely used in the semiconductor industry to determine the resistivity of uniform samples (References 3 and 4). As originally devised by van der Pauw, one uses an arbitrarily shaped (but simply connected, i.e., no holes or nonconducting islands or inclusions) thin-plate sample containing four very small ohmic contacts placed on the periphery (preferably in the corners) of the plate. A schematic of a rectangular van der Pauw configuration is shown in Fig. 2.

The objective of the resistivity measurement is to determine the sheet resistance \( R_s \). Van der Pauw demonstrated that there are actually two characteristic resistances \( R_A \) and \( R_B \) associated with the corresponding terminals shown in Fig. 2. \( R_A \) and \( R_B \) are related to the sheet resistance \( R_S \) through the van der Pauw equation

\[
\exp(-\pi R_A/R_S) + \exp(-\pi R_B/R_S) = 1
\]

which can be solved numerically for \( R_s \).

The bulk electrical resistivity \( \rho \) can be calculated using

\[
\rho = R_Sd.
\]
In order to measure the two characteristic resistances $R_A$ and $R_B$, one applies a constant dc current $I$ into contact 1 and out of contact 2, while measuring a voltage $V_{43}$ from contact 4 to contact 3 as shown in Fig. 2. Next, one applies the current $I$ into contact 2 and out of contact 3 while measuring a voltage $V_{14}$ from contact 1 to contact 4. $R_A$ and $R_B$ are calculated by means of the following expressions:

$$R_A = \frac{V_{43}}{I_{12}} \quad \text{and} \quad R_B = \frac{V_{14}}{I_{23}}.$$  (5)

The objective of the Hall measurement in the van der Pauw technique is to determine the sheet carrier density $n_s$ by measuring the Hall voltage $V_H$. The Hall voltage measurement consists of a series of voltage measurements with a constant current $I$ and a constant magnetic field $B$ applied perpendicular to the plane of the sample. Conveniently, the same sample, shown again in Fig. 3, can also be used for the Hall measurement. In order to measure the Hall voltage $V_H$, a current $I$ is forced through an opposing pair of contacts 1 and 3, and the Hall voltage $V_H$ ($= V_{24}$) is measured across the remaining pair of contacts 2 and 4. Once the Hall voltage $V_H$ is obtained from the Hall measurement, the sheet carrier density $n_s$ can be calculated via $n_s = \frac{IB}{q|V_H|}$ from the known values of $I$, $B$, and $q$.

One should keep in mind that the sample must be uniform in terms of its electrical properties and its thickness, and that the accuracy of determination of the free-carrier density is typically limited by the conducting layer thickness measurement. On the other hand, the accuracy of the free-carrier mobility is typically determined by that of the magnetic field measurement. One further assumes that the sample lateral dimensions are large compared to the ohmic contacts and the sample thickness.

In practical situations there are several departures from the ideal case which must be considered when carrying out Hall and resistivity measurements. Primary concerns are (1) ohmic contact electrical quality and size, (2) sample uniformity and accurate conducting layer thickness determination, and (3) thermo-galvanomagnetic effects due to nonuniform temperature. Also, one must accurately measure sample temperature, magnetic field intensity, electrical current, and voltage.
III. Resistivity and Hall Measurements

Sample Geometry
Definitions for Resistivity Measurements
Resistivity Measurements
Resistivity Calculations
Definitions for Hall Measurements
Hall Measurements
Hall Calculations

The following procedures for carrying out Hall measurements provide a guideline for the beginning user who wants to learn operational procedures, as well as a reference for experienced operators who wish to invent and engineer improvements in the equipment and methodology.

Sample Geometry

It is preferable to fabricate samples from thin plates of the semiconductor material and to adopt a suitable geometry, as illustrated in Fig. 4. The average diameters \( D \) of the contacts, and sample thickness \( d \) must be much smaller than the distance between the contacts \( L \). Relative errors caused by non-zero values of \( D \) are of the order of \( D/L \).

The following equipment is required:

- Permanent magnet, or an electromagnet (500 to 5000 gauss)
- Constant-current source with currents ranging from 10 \( \mu \)A to 100 mA (for semi-insulating GaAs \( \rho \approx \text{lo}^7 \) \( \Omega \text{cm} \), a range as low as 1 nA is needed)
- High input impedance voltmeter covering 1 \( \mu \)V to 1 V
- Sample temperature-measuring probe (measuring a tenth of a degree Celsius for high accuracy work)

Definitions for Resistivity Measurements

Four leads are connected to the four ohmic contacts on the sample. These are labeled 1, 2, 3, and 4 counter-clockwise as shown in Fig. 4a. It is important to use the same batch of wire for all four leads in order to minimize thermoelectric effects. Similarly, all four ohmic contacts should consist of the same material.

Here, we define the following parameters. See Fig. 2.

\( \rho \) = sample resistivity (in \( \Omega \text{cm} \))

\( d \) = conducting layer thickness (in cm)

\( I_{12} \) = positive constant dc current \( I \)

For \( \sum_{i=1}^{I} I_{i} \) (in amperes, A)
$V_{12} = \text{dc voltage measured between contacts 1 and 2 (} V_1 - V_2 \text{) without applied magnetic field (0). Likewise for } V_{23}, V_{34}, V_{41}, V_{21}, V_{43}, V_{32} \text{ (in volts, V)}$

**Resistivity Measurements**

The data must be checked for internal consistency, for ohmic contact quality, and for sample uniformity.

- Set up a dc current $I$ such that when applied to the sample the power dissipation does not exceed 5 mW (preferably 1 mW). This limit can be specified before the automatic measurement sequence is started measuring the resistance $R$

  $$I < (200R)^{-0.5}.$$  

- Apply the current $I_{21}$, and measure voltage $V_{34}$.
- Reverse the polarity of the current ($I_{12}$) and measure $V_{43}$.
- Repeat for the remaining six values ($V_{41}, V_{14}, V_{12}, V_{21}, V_{23}, V_{32}$).

Eight measurements of voltage yield the following eight values of resistance, all of which must be positive:

$$
\begin{align*}
R_{21,34} &= V_{34}/I_{21}, \\
R_{32,41} &= V_{41}/I_{32}, \\
R_{43,12} &= V_{12}/I_{43}, \\
R_{14,23} &= V_{23}/I_{14}, \\
R_{12,43} &= V_{43}/I_{12}, \\
R_{23,14} &= V_{14}/I_{23}, \\
R_{34,21} &= V_{21}/I_{34}, \\
R_{41,32} &= V_{32}/I_{41}.
\end{align*}
$$

These are all that are needed to determine the sheet resistance $R_s$. If the conducting layer thickness $d$ is known, the bulk resistivity $\rho$ is $R_s d$.

Because this sequence of measurements is redundant, it permits important consistency checks on measurements, ohmic contact quality, and sample uniformity.

**Resistivity Calculations**

- Consistency requires that all eight $R$ values be positive. If any $R$ value fails this test, identify the source of error.
- Contact quality requires that:

  $$
\begin{align*}
R_{21,34} &= R_{12,43}, \\
R_{32,41} &= R_{23,14}, \quad \text{and} \quad R_{43,12} &= R_{34,21}, \\
R_{14,23} &= R_{41,32}.
\end{align*}
$$

If any of these four tests fail to be true within 5% (preferably 3%), investigate the sources of error.

- Uniformity of the sample’s electrical properties requires that:

  $$
\begin{align*}
R_{21,34} + R_{12,43} &= R_{43,12} + R_{34,21}, \quad \text{and} \\
R_{32,41} + R_{23,14} &= R_{14,23} + R_{41,32}.
\end{align*}
$$

If either of these two tests fails to be true within 5% (preferably 3%), investigate the sources of error.
The sheet resistance $R_S$

\begin{align*}
R_A &= \frac{(R_{21,34} + R_{12,43} + R_{43,12} + R_{34,21})}{4}, \quad \text{and} \\
R_B &= \frac{(R_{32,41} + R_{23,14} + R_{14,23} + R_{41,32})}{4} \quad (10)
\end{align*}

via the van der Pauw equation [Eq. (3)]. For numerical solution of Eq. (3), see the routine in Section IV. If conducting layer thickness $d$ is known, the bulk resistivity $\rho = \frac{R_S}{d}$ can be calculated from $R_S$.

**Definitions for Hall Measurements**

The Hall measurement, carried out in the presence of a magnetic field, yields the free-carrier sheet density $I$ and hence the bulk carrier density $n$ or $p$ (for $n$-type or $p$-sample is known. The Hall voltage for thick, heavily doped samples can be quite small (of the order of microvolts).

The difficulty in obtaining accurate results is not merely the small magnitude of the Hall voltage itself, since good quality digital voltmeters on the market today are quite adequate. The more severe problem comes from the large offset voltage caused by non-symmetric contact placement, sample shape, and non-uniform temperature.

The most common way to control this problem is to acquire two sets of Hall measurements, one for positive and one for negative magnetic field direction. The relevant definitions are as follows (Fig. 3):

- $I_{13}$ = dc current injected into lead 1, and taken out of lead 3. Likewise for $I_{31}$, $I_{42}$, $I_{24}$.
- $B$ = constant and uniform magnetic field intensity (to within 3%) applied parallel to the z-axis within a few degrees (Fig. 3). $B$ is positive when pointing in the positive z direction, and negative when pointing in the negative z direction.
- $V_{24P} = \text{dc Hall voltage measured between leads 2 and 4 with magnetic field positive for } I_{13}$. Likewise for $V_{42P}$, $V_{13P}$, $V_{31P}$.
- Similar definitions for $V_{24N}$, $V_{42N}$, $V_{13N}$, $V_{31N}$ apply when the magnetic field $B$ is reversed.

**Hall Measurements**

The procedure for the Hall measurement is:

- Apply a positive magnetic field $B$.
- Apply a positive magnetic field $B$.
- Apply a dc current $I_{13}$ to leads 1 and 3, and measure $V_{24P}$.
- Apply a dc current $I_{31}$ to leads 3 and 1, and measure $V_{42P}$.
- Likewise, measure $V_{13P}$ and $V_{31P}$ with $I_{42}$ and $I_{24}$, respectively.
- Reverse the magnetic field (negative $B$).
- Likewise, measure $V_{24N}$, $V_{42N}$, $V_{13N}$, and $V_{31N}$ with $I_{13}$, $I_{31}$, $I_{42}$, and $I_{24}$, respectively.
The above eight measurements of Hall voltages $V_{24P}, V_{42P}, V_{13P}, V_{31P}, V_{24N}, V_{42N}, V_{13N}, V_{31N}$ are all that are needed to determine the sample type ($n$ or $p$) and the sheet carrier density $n_s$. The Hall mobility can be determined from the sheet density $n_s$ and the sheet resistance $R_S$ obtained in the resistivity measurement. Solve Eq. (2).

This sequence of measurements provides redundancy and may allow a consistency check to determine if the carrier density is graded rather than constant across the sample.

**Hall Calculations**

Before calculating Hall results, the voltages must be corrected for offset and tested for carrier density non-uniformity. To do this,

- Calculate the following (be careful to maintain the signs of measured voltages):

  \[ V_C = V_{24P} - V_{24N}, \quad V_D = V_{42P} - V_{42N}, \quad V_E = V_{13P} - V_{13N}, \quad \text{and} \quad V_F = V_{31P} - V_{31N}. \]  

  \[ \text{(11)} \]

- If the measurements are not influenced by gradients across the sample, then

  \[ V_C = V_D \quad \text{and} \quad V_E = V_F. \]

  \[ \text{(12)} \]

If this test fails to be true within 5\% (preferably 3\%), investigate the sources of error.

- The sample type is determined from the polarity of the voltage sum $V_C + V_D + V_E + V_F$.

  If this voltage sum is positive (negative), the sample is $p$-type ($n$-type).

- The sheet-carrier density (in units of $\text{cm}^{-2}$) is calculated from

  \[ n_s = 8 \times 10^{-8} IB/[q(V_C + V_D + V_E + V_F)] \]

  if the voltage sum is positive, or

  \[ n_s = |8 \times 10^{-8} IB/[q(V_C + V_D + V_E + V_F)]| \]

  if the voltage sum is negative,

  \[ \text{(13)} \]

  where $B$ is the magnetic field in gauss (G) and $I$ is the dc current in amperes (A).

- The bulk density can be determined if the conducting layer thickness $d$ of the sample is known:

  \[ n = n_s/d \]

  \[ p = p_s/d \]

  \[ \text{(14)} \]

- The Hall mobility $\mu = 1/qn_s R_S$ (in units of $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$) is calculated from the sheet carrier density $n_s$ ($p_s$) and the sheet resistance $R_s$. See Eq. (2).

The procedure for this sample is now complete. The final printout might contain (Sample Hall Worksheet):

- Sample identification, such as ingot number, wafer number, sample geometry, sample temperature, thickness, data, and operator.
- Values of dc current I and magnetic field B.
- Calculated value of sheet resistance $R_S$, and resistivity if thickness $d$ is known.
- Calculated value of sheet carrier density $n_s$ or $p_s$, and the bulk-carrier density $n$ or $p$ if $d$ is known.
- Calculated value of Hall mobility $\mu$.
Sample geometries for van der Pauw resistivity and Hall effect measurements. The cloverleaf design will have the lowest error due to finite contact size, but it is more difficult to fabricate than a square or rectangle.
Schematic of a van der Pauw configuration used in the determination of the two characteristic resistances $R_A$ and $R_B$.

$$R_A = \frac{V_{43}}{I_{12}}$$

$$R_B = \frac{V_{14}}{I_{23}}$$
Sources of Error

Check the following:

1. Are the contact $I-V$ characteristics linear?
2. Is there visible damage (cracks, especially around the contacts)?
3. Is the sample in the dark?
4. Is the sample temperature uniform?
5. If there are large temperature gradients across the wiring, are dissimilar wiring materials being used?
Figure 3

Schematic of a van der Pauw configuration used in the determination of the Hall voltage $V_H$.

Last edited May 22, 2000
IV. Algorithm Example

The sheet resistance $R_S$ can be obtained from the two measured characteristic resistances $R_A$ and $R_B$ by numerically solving the van der Pauw equation [Eq. (3) in the text] by iteration

$$\exp(-\pi R_A/R_S) + \exp(-\pi R_B/R_S) = 1$$

as outlined in the following routine:

Set the error limit $\delta = 0.0005$, corresponding to 0.05 %

Calculate the initial value of $z_1$, or $z_0 = 2 \ln(2)/(\pi (R_A + R_B)]$

Calculate the $i^{th}$ iteration of $y_i = 1/\exp(\pi z_{i-1} R_A) + 1/\exp(\pi z_{i-1} R_B)$

Calculate the $i^{th}$ iteration of $z_i$ where

$$z_i = z_{i-1} - [(1-y_i)/\pi] / [R_A/\exp(\pi z_{i-1} R_A) + R_B/\exp(\pi z_{i-1} R_B)]$$

When $(z_i - z_{i-1})/z_i$ is less than 6, stop and calculate the sheet resistance $R_S = 1/z_i$

The resistivity $\rho$ is given by $\rho = R_S d$, where $d$ is the thickness of the conducting layer.

Last edited May 22, 2000
Van der Pauw Hall Measurement Worksheet

Sample Identification
Thickness if known (cm)
Date
Lab
Chemical Pretreatment (if any)
Contact Metal
Comments

Resistivity Measurement

\[
\begin{align*}
& \{ I_{21} \quad V_{34} \quad R_{21,34} \} \\
& \{ I_{12} \quad V_{43} \quad R_{12,43} \} \\
& \{ I_{32} \quad V_{41} \quad R_{32,41} \} \\
& \{ I_{23} \quad V_{14} \quad R_{23,14} \} \\
& \{ I_{43} \quad V_{12} \quad R_{43,12} \} \\
& \{ I_{34} \quad V_{21} \quad R_{34,21} \} \\
& \{ I_{14} \quad V_{23} \quad R_{14,23} \} \\
& \{ I_{41} \quad V_{32} \quad R_{41,32} \}
\end{align*}
\]

Temperature (°C or K)

\[
R_A = \quad \text{(Eq. 10)}
\]

\[
R_B = \quad \text{(Eq. 10)}
\]

\[
\exp(-\frac{\pi}{2} \frac{R_A}{R_S}) + \exp(-\frac{\pi}{2} \frac{R_B}{R_S}) = 1
\] (Eq 3, section IV algorithm, or see ASTM F76)

\[
R_S = \quad \Omega/\square
\]

For known thickness:
\[
\rho = R_S d
\]

Hall Voltage Measurement

\[
\begin{align*}
& +B \text{ Field (G)} \\
& I_{13} \quad V_{24P} \\
& I_{31} \quad V_{42P} \\
& I_{24} \quad V_{31P} \\
& -B \text{ Field (G)} \\
& I_{13} \quad V_{24N} \\
& I_{31} \quad V_{42N} \\
& I_{24} \quad V_{31N}
\end{align*}
\]

Temperature (°C or K)

\[
\sum V_i
\] (Eq. 11)

\[
n_S = 8 \times 10^{-3} \frac{I_B}{q |\Sigma V_i|} \quad (\text{Eq. 13})
\]

\[
n_S = \quad \text{cm}^2
\]

For known thickness:
\[
n = n_s / d
\]

\[
\mu = 1 / q n_S R_S = \quad \text{cm}^2 \text{V}^{-1} \text{s}^{-1} \quad (\text{Eq. 2})
\]
V. References


