The circular loop

Let us calculate the magnitude and direction of \( B \) on the axis of a circular loop of radius \( a \) carrying a current \( I \), as in Figure 7-4.

An element \( I \, dl \) of current produces a magnetic induction \( dB \), as indicated in the figure. By symmetry, the total magnetic induction will be along the axis, and

\[
dB_z = \frac{\mu_0 I \, dl}{4\pi r^2} \cos \theta \quad (7-18)
\]

hence

\[
B_z = \frac{\mu_0 I 2\pi a \cos \theta}{4\pi r^2} = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \quad (7-19)
\]

The magnetic induction is maximum in the plane of the ring and drops off as \( z^3 \) for \( z^2 >> a^2 \).

7.3 The force on a point charge moving in a magnetic field

Equation 7-7 gave us the force on a closed circuit immersed in a magnetic field; it can also give us the force on a single charge \( Q \) moving at a velocity \( v \) in a magnetic field \( B \). We shall see that Eq. 7-7 is consistent with the Lorentz force we have already found in Eq. 6-8.

The force on a current element \( I \, dl \) is \( I \, dl \times B \). Now, if the cross-sectional area of the wire is \( da \),

\[
I = n(da \, v)Q \quad (7-20)
\]

where \( n \) is the number of carriers per unit volume, \( v \) is their average drift velocity, and \( Q \) is the charge on one carrier. The reason for this relation is that the total charge flowing per second is the charge on the carriers that are contained in a length \( v \) of the wire.

Then the force on the element \( dl \) is

\[
n \, da \, dl \, Q \, v \times B,
\]

and the force on a single charge \( Q \) moving at a velocity \( v \) in a field \( B \) is

\[
Q \, v \times B.
\]

This force is perpendicular both to the velocity \( v \) and to the local magnetic induction \( B \).

More generally, if there is also an electric field \( E \), the force is

\[
Q[E + (v \times B)].
\]

This is the Lorentz force.
**Example**

**HALL EFFECT IN SEMICONDUCTORS**

We have already seen in the example on page 176, that semiconductors contain either or both of two types of mobile charges, namely conduction electrons and holes. When a current flows through a bar of semiconductor in the presence of a transverse magnetic field, as in Figures 7-5a or 7-5b, the mobile charges drift, not only in the direction of the applied electric field, but also in a direction perpendicular to both the applied electric and magnetic fields. This gives rise to a voltage difference between the upper and lower electrodes.

If the voltmeter connected to these electrodes draws a negligible amount of current, the plates charge up until their field $E_t$ is sufficient to stop the transverse drift. This transverse electric field is called
the Hall field. The net transverse force on the mobile charges inside the bar is then zero as in Figures 7-5c and 7-5d.

Let us assume that the conduction is due to holes carrying charges \( +e \), \( e \) being the magnitude of the electronic charge, and that there are \( p \) holes per cubic meter. When the transverse drift has stopped,

\[
eE_t = e \frac{V}{b} = evB. \tag{7-21}
\]

Now

\[
I = Jab = (peva)b, \tag{7-22}
\]

and

\[
V = \frac{1}{pea} IB. \tag{7-23}
\]

If the conduction is due rather to conduction electrons carrying a charge \( -e \), the Hall field has the opposite polarity, as in Figure 7-5b.

Note that in both cases the carriers are swept down by the magnetic field.

Although the Hall effect is in fact more complex than we have assumed it to be, our value of \( V \) is nonetheless approximately correct. The Hall effect is commonly used for measuring magnetic fields and for studying conduction phenomena in semiconductors.

Example

**THE HODOSCOPE**

The hodoscope is a device that simulates the trajectory of a charged particle in a magnetic field. The principle involved is quite simple: if the charged particle, of mass \( m \), charge \( Q \), and velocity \( v \), is replaced by a light wire fixed at the two ends of the trajectory and carrying a current \( I \), the wire will follow the trajectory if

\[
\frac{mv}{Q} = \frac{T}{I}, \tag{7-24}
\]

where \( T \) is the tension in the wire. This statement is by no means obvious, and we shall have to demonstrate its validity.

The advantage of the so-called floating wire lies in the fact that it is much easier to experiment with a wire than with an ion beam; the wire is, in effect, an analog computer.

Let us first consider a region where the ion beam is perpendicular to \( B \) as in Figure 7-6a. Then

\[
BQv = \frac{mv^2}{R_t}, \tag{7-25}
\]

where \( R_t \) is the radius of curvature of the trajectory:

\[
R_t = \frac{mv}{BQ}. \tag{7-26}
\]