PLASMA PHENOMENA IN GAS DISCHARGES

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3.4. The Hall effect in the positive column

The effect of a transverse magnetic field on a positive column depends on its magnitude. If it is sufficiently large all particles are constrained by it to make collisions with the wall, and the effect is gross with virtual cathodes forming on the wall (see Francis 1957, p. 169); however, for weak fields it is possible to treat the problem by the methods of the last two chapters and also to make experimental measurements.

For simplicity consider a plane model with the magnetic field in the y-direction and the column axis in the z-direction. Only the electrons are assumed to be influenced by the magnetic field and their equation of motion becomes, for $\nu_e \gg Z$,

$$mv_e v_e = -e(E + v_e \times B) - \frac{1}{n} \nabla(nk_B T_e),$$

which separates into

$$mv_e v_{ez} = -eE_z$$

and

$$mv_e v_{ex} = -e(E_x - v_{ez}B_y) - \frac{k_B T_e}{n} \frac{dn}{dx}.$$

Ignoring electron inertia and combining with the ion equation of motion $(T_i = 0)$,

$$M \left( v_{ix} \frac{dv_{ix}}{dx} + Zv_{ix} + v_{ix}v_{ix} \right) = eE_x$$

and the continuity equation,

$$\frac{d}{dx} (nv_{ix}) = \frac{d}{dx} (nv_{ex}) = Zn,$$

gives normalized equations

$$\frac{d\xi}{du} = \frac{1 - u^2}{1 + u^2 \delta + u\varepsilon_H}, \quad (3.18)$$

$$\frac{d\eta}{du} = \frac{(1 + \delta) + \varepsilon_H u^2}{1 + u^2 \delta + \varepsilon_H u}, \quad (3.19)$$

where the normalizations are as previously and in addition

$$\varepsilon_H = -\frac{ev_{ez}B}{Z(k_B T_e M)^{1.5}}.$$

The distributions cease to be symmetric about the midplane; in physical terms this is because of the $j \times B$ force due to the discharge current.
Fig. 3.8. The Hall coefficient $R_H = \alpha_{H}/e$ of a positive column as a function of the collision parameter $v_i/Z$ in cylindrical and plane geometry. Measurements of $\alpha_{H}$ as a function of pressure in helium and nitrogen compared with theory.
Now the Hall coefficient in this situation can conveniently be defined in terms of the difference $\Delta V_H$ in potential at opposite points on the walls by

$$\Delta V_H = \alpha_H \cdot 2L \cdot v_{ez} \cdot B.$$  \hspace{1cm} (3.20)

The Hall coefficient $R_H$ is then

$$R_H = \frac{\Delta V_H}{2L} = \frac{2L}{\int nev_{ez} \, dx \cdot B} = \frac{\alpha_H}{\bar{n}e}.$$  

Treating $\varepsilon_H$ as a small parameter $< 1$, $\xi$ and $\eta$ can be expanded as power series in $\varepsilon_H$,

$$\xi = \xi_0(u) + \varepsilon_H \xi_1(u) + \varepsilon_H^2 \cdots,$$

$$\eta = \eta_0(u) + \varepsilon_H \eta_1(u) + \varepsilon_H^2 \cdots,$$

and then to lowest order in $\varepsilon_H$, $\alpha_H$ is $\eta_1(1)/\xi_0(1)$. Similar expressions result in cylindrical geometry. The free-fall model of the positive column can also be analysed in the same spirit, and results have been given by Ecker and Kanne (1964).

Fig. 3.8 shows $\alpha_H$ as a function of $v/Z$; a comparison is also made of the values measured by Anderson (1964) with theory for nitrogen and helium over a range of pressure multiplied by radius.