and therefore
\[ \delta^{(1)} = \frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_2} = 4.688 \pm 0.13 \]
\[ \delta^{(2)} = \frac{\lambda_3 - \lambda_2}{\lambda_4 - \lambda_3} = 4.117 \pm 0.49. \]

These results are consistent with the asymptotic value given in Eq. (3.25), even though input from only the first four bifurcations was used.

The determination of the second Feigenbaum number \( \alpha \) is not possible with the present data. As pointed out previously, the intervals \( \Delta y_n^* \) must be selected appropriately, but even then (see Fig. 3.36) the ratios of \( \Delta y_n^* \) seem much larger than \( \alpha \). This is due in part to the fact that one has not reached the asymptotic regime of Eq. (3.26) and in part to discontinuous jumps in \( V_b \) at certain values\(^{15} \) of \( V_0 \). However, it is evident from the data that the system replicates itself after each bifurcation. Furthermore, the spacing between stable points in every branch decreases in subsequent bifurcations by a multiplicative factor; this factor seems to converge toward Feigenbaum's \( \alpha \). We also note that for the 1N4001 diode it was possible to observe islands of stability in the chaotic region.

3.8. LOCK-IN DETECTION

Suppose one is studying a signal, amid noise, that comes at a specific frequency. We can use this to pick the signal out of the noise. Furthermore, we can be sensitive to the phase of the signal as well as its frequency, and that can make a huge improvement. The technique that does all this is called phase-sensitive detection. The device that you do it with is called a lock-in amplifier.

There are two inputs to a lock-in amplifier. One input carries the signal (and the noise). The signal, remember, is varying at some specific frequency which you are aware of. It may be completely buried in noise, however, so you would not see it on an oscilloscope, for example. The other input carries a reference that varies at the frequency of the signal. The signal oscillates because you make it do so, and the way you do that also gives you the reference. For example, your experiment measures a response to a

\(^{15}\)Some diodes show marked hysteresis associated with these discontinuities.
3.8 Lock-In Detection

laser, so you turn the laser on and off rapidly with a mechanical chopper. The motor drive for the chopper gives you the reference signal.

The lock-in amplifier takes the reference signal and uses it as a switch. For half the period, the switch is “up,” and it lets the signal input pass through it with no change. For the other half, the switch is “down,” and it reverses the sign of the signal (i.e., multiplies it by \(-1\)) before it passes. This is shown in Fig. 3.37. The result of this is a modified signal that is always positive, instead of oscillating around 0 like the input signal. A low-pass filter takes out the remaining oscillation and lets the DC level pass through. This DC level is read off a meter, presented at some output connector, or digitized by some computer, depending on the lock-in amplifier.

Now consider what happens if the signal is out of phase by 90° with respect to the reference. This situation is shown in Fig. 3.38. Now the output of the multiply stage is still something that oscillates about 0. The average DC level is 0, and that is the output of the lock-in amplifier. So, as promised, the lock-in amplifier only detects signals that are in phase with the reference. Most lock-ins have a “phase adjustment” knob on the front that allows you to maximize the output signal. If you have the phase 180° away, the output signal should reverse sign.

Now consider what the lock-in amplifier does to noise that has some frequency other than the frequency of the signal. The answer is obvious. The output of the multiply stage will just be a jumble of noise like the input stage since the reference is essentially just randomly flipping amplitudes.
The output of the low-pass filter will *average* to 0 over some time determined by the $RC$ time constant of the filter.

The lock-in amplifier is actually quite a versatile instrument. One of its uses beyond noise rejection is as a spectroscopy tool. Let’s say you have a signal $y$ that is a function of some parameter $x$. For example, you might have an NMR signal as a function of the large magnetic field that polarizes the sample. Such a thing is graphed in Fig. 3.39. Now assume the signal
is modulated (i.e., made to oscillate) by setting $x$ to some central value $x_0$ and making it oscillate about $x_0$ by a small amount $\Delta x$. Then the amplitude $\Delta y$ of the modulated signal is given by

$$\Delta y = \left. \frac{dy}{dx} \right|_{x_0} \Delta x.$$ 

In other words, the output of the lock-in is the derivative of the line shape $y(x)$. It does this, of course, while throwing out any noise that gets in its way. One common technique, described in detail by Dunlap (1988), is to sweep the value of $x$ many times and record the output in a multi-channel analyzer. This uses signal averaging to get rid of any remaining noise.

### 3.9. COMPUTER INTERFACES

Many of the experiments described in this book, as well as in many undergraduate instructional laboratories, can be done without the use of sophisticated computerized data acquisition. Indeed, in experiments such as the Balmer series in hydrogen (Section 1.5.3), the Faraday effect (Section 5.7), and the $\gamma-\gamma$ angular correlation in $^{60}$Co (Section 9.5.4), for example, there is much instructional value in taking, recording, and analyzing data “by hand.”

Nevertheless, directly interfacing a computer with the experiment makes it possible to take data much more quickly in many cases, and this also has much instructional value. Furthermore, some experiments that had once been very difficult, if not impossible, in the instructional laboratory, can now be done with relatively simple and inexpensive computer interfaces. A wide variety of commercial interfaces exist, and it is not possible to cover all of them in this textbook. Indeed, the market moves quickly and different options appear and disappear very regularly. A recent publication, available free from Keithley at [http://www.keithley.com/](http://www.keithley.com/), is the “Data Acquisition and Control Handbook.” However, a number of standard situations apply.

The simplest computer interface is a “serial” interface using an RS232 standard communications port on the computer. The electronics on your computer and in the data acquisition device to which you wish to interface support a standard “handshake” protocol for moving instructions and data back and forth between the two devices. All that you need is