(b) For angles \( \theta > \pi/2 \) the cross section does not vary as rapidly, and at the limit \( \theta \approx \pi \) it has the value \( d\sigma/d\Omega = k = 1.2 \times 10^{-24} \text{ cm}^2 \). Precise measurements at these large angles reveal deviations from the Rutherford equation and are due to the strong interaction (nuclear interaction, see Table 6.1) between the alpha particle and the gold nucleus.

As explained in detail in the introduction to this chapter, the nuclear interaction will manifest itself only at short distances— that is, at high-momentum transfers \( q \) (Eq. 1.4), where \( q = 2p \sin \theta/2 \). Clearly the maximum momentum transfer in this experiment is

\[
q = 2p = 2 \sqrt{2Em} = 200 \text{ MeV}/c
\]

The recent experiments on Coulomb scattering of electrons from protons have been extended to \( q \approx 2.2 \text{ BeV}/c \).

3. Compton Scattering

3.1 Frequency Shift and Cross Section

This section deals with the scattering of electromagnetic radiation by free electrons. As mentioned in the introduction to this chapter, it is the scattering of electromagnetic radiation from various objects that makes it possible for us to "see" them. However, as the frequency of the radiation is increased beyond the visible region, the light quanta have energies comparable to, or larger than the binding energy of the electrons in atoms, and the electrons can therefore be considered as free.

In 1920 A. H. Compton investigated the scattering of monochromatic x-rays from various materials. He observed that after the scattering, the energy (frequency) of the x-rays had changed, and had always decreased. From the point of view of classical electromagnetic theory, this frequency shift cannot be explained, since the frequency is a property of the incoming electromagnetic wave (field) and cannot be altered by the change of direction implied by the scattering. If, on the other hand, we think of the incoming radiation as being represented by a beam of photons, we need only consider the scattering of a quantum of energy \( E = \hbar \nu \) from a free electron; then, because of energy-momentum conservation, the scattered quantum has energy \( E' = \hbar \nu' < E \), in complete agreement with the experiments of Compton.

The frequency shift will depend on the angle of scattering and can be easily calculated from the kinematics. Consider an incoming photon of energy \( E = \hbar \nu \) and momentum \( p = \hbar \nu/c \) (Fig. 6.8) scattering from an electron of mass \( m \); \( p \) is the momentum of the electron after scattering and

\[ \text{See, for example, J. D. Jackson, Classical Electrodynamics, John Wiley, p. 488.} \]
3. Compton Scattering

FIG. 6.8 Compton scattering of a photon from a free electron.

\( h\nu', h\nu'/c \) the energy and momentum of the photon after the scattering. The three vectors \( h\nu/c, h\nu'/c, \) and \( p \) must lie on the same plane, and energy conservation yields

\[
h\nu + mc^2 = h\nu' + \sqrt{p^2c^2 + m^2c^4}
\]

(3.1)

From momentum conservation we obtain

\[
h\nu = h\nu' \cos \theta + cp \cos \phi
\]

(3.2)

\[0 = h\nu' \sin \theta + cp \sin \phi
\]

(3.3)

Here \( \theta \) is the photon scattering angle, and \( \phi \) the electron recoil angle. To solve the above equations we transpose appropriately, square, and add Eq. 3.2 and Eq. 3.3 to obtain

\[h^2\nu^2 - 2h^2\nu' \cos \theta + h^2\nu'^2 = c^2p^2
\]

while by squaring Eq. 3.1,

\[h^2\nu^2 + h^2\nu'^2 - 2h^2
\nu' + 2hmc^2(\nu - \nu') = c^2p^2
\]

which by subtraction yields

\[
\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{mc^2} (1 - \cos \theta)
\]

(3.4)

We can recast Eq. 3.4 into two more familiar forms: (a) to give the shift in wavelength of the scattered x-ray beam:

\[\Delta \lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)
\]

(3.5)

or (b) to give the energy of the scattered photon:

\[E' = \frac{E}{1 + (E/mc^2)(1 - \cos \theta)}
\]

(3.6)

From Eq. 3.5 we see that the shift in wavelength, except for the angular
dependence, is a constant, the Compton wavelength\[ h/mc \] = 2.42 \times 10^{-10} \text{ cm} = 0.0242 \text{ Å} 

For low-energy photons, with \( \lambda \gg 0.02 \text{ Å} \), the Compton shift is very small, whereas for high-energy photons with \( \lambda \ll 0.02 \text{ Å} \), the wavelength of the scattered radiation is always of the order of 0.02 Å, the Compton wavelength. These conclusions can equally well be obtained from Eq. 3.6, where the energy shift increases when \( E/mc^2 \) becomes large. For \( E/mc^2 \gg 1 \), \( E' \) is independent of \( E \) and of the order \( E' \approx mc^2 \). [Hence \( \lambda' = c/v' = c/(E'/h) \approx c/(mc^2/h) = h/mc \) as stated before.]

As an example, in this laboratory gamma rays from Cs\(^{137} \) are scattered from an aluminum target; since \( E = 0.662 \text{ MeV} \), we have \( E/mc^2 = 1.29 \), so that back-scattered gamma rays (\( \theta = 180^\circ \)) will have \( E' = E/3.6 \), which is less than 30 percent of their original energy. It thus becomes quite easy to observe the Compton energy shift as compared to x-ray scattering, where, if we assume \( \lambda = 2 \text{ Å} \), \( \Delta\lambda/\lambda = \Delta E/E = 0.01 \).

In the original experiments Compton and his collaborators observed (especially for high \( Z \) materials) in addition to the frequency shifted x-rays, scattered radiation not shifted in frequency. The unshifted x-rays are due to scattering from electrons that remained bound in the atom\[‡ \] in this process the recoiling system is the entire atom; and we replace in Eq. 3.5 \( m \) by \( m_A \) (where \( m_A \approx 2,000 \times A \times m_e \)) resulting in an undetectable wavelength shift, \( \Delta\lambda' \approx 10^{-7} \text{ Å} \).

Next we are interested in the differential cross section for the scattering of the radiation from the electrons. Classically this is given by the Thomson cross section,\[§ \] which can be easily derived: consider a plane wave propagating in the \( z \) direction with the \( E \) vector linearly polarized along the \( x \) direction. This is incident on an electron of mass \( m \), as shown in Fig. 6.9. The electron will experience a force \( F = eE = eE_0 \cos \omega t \), and its acceleration will be \[ \dot{v} = \frac{eE_0}{m} \cos \omega t \]

\[ \text{Fig. 6.9 Classical picture of the scattering of electromagnetic radiation by an electron; this leads to the Thomson cross section.} \]

\[ \text{† The mass of the electron } m_e \text{ was used in evaluating } h/mc; \text{ by using the mass of the pion, or other particle, we obtain the pion Compton wavelength, and so forth.} \]

\[ \text{‡ A similar situation is discussed in the following section on the Mössbauer effect, where the nucleus remains bound in the lattice and the recoiling system is the entire crystal.} \]

\[ \text{§ Already discussed in Chapter 5, Section 2.5.} \]
According to Eq. 2.26, Chapter 5, the power radiated by this accelerated electron will be (nonrelativistically, in MKS units)

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi\varepsilon_0)^2 \mu_0 c^5} \hat{\theta}^2 \sin^2 \Theta$$

(3.7)

where $\Theta$ is the angle between the direction of observation and the $E$ vector of the incoming wave. Using the expression for $\hat{\theta}$, we can write for Eq. 3.7 averaged over one cycle

$$\langle \frac{dP}{d\Omega} \rangle = \frac{1}{2} \left( \frac{e^2}{4\pi\varepsilon_0 mc^2} \right)^2 \varepsilon_0 E_0^2 c \sin^2 \Theta$$

Finally, from the definition for a cross section given in Chapter 5, Section 2.1, we have

$$\frac{d\sigma}{d\Omega} = \frac{\text{energy radiated/(unit time-unit solid angle)}}{\text{incident energy/(unit area-unit time)}}$$

Here the denominator is clearly given by the Poynting vector

$$\langle P \rangle = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 = \frac{1}{2} \varepsilon_0 c E_0^2$$

Thus we obtain

$$\frac{d\sigma}{d\Omega} = \left( \frac{e^2}{4\pi\varepsilon_0 mc^2} \right)^2 \sin^2 \Theta$$

(3.8)

Where

$$\frac{e^2}{4\pi\varepsilon_0 mc^2} = r_0$$

has dimensions of length, the so-called "classical electron radius"

$$r_0 = 2.82 \times 10^{-13} \text{ cm}$$

Finally, we average over all possible directions of polarization of the incoming wave and use the angle $\theta$ measured from the direction of propagation of the wave to obtain

$$\frac{d\sigma}{d\Omega} = r_0^2 \left( \frac{1 + \cos^2 \theta}{2} \right) \text{cm}^2$$

(3.9)

When integrated over all angles, Eq. 3.9 yields the Thomson cross section

$$\sigma_T = \frac{8\pi}{3} r_0^2$$

(3.10)

(also given in Chapter 5, Eq. 2.19).
Several objections can be raised to the simple cross section given by Eq. 3.9 or Eq. 3.10: (a) it does not depend on frequency, a fact not supported by experiment; (b) the electron, even though free, is assumed not to recoil; (c) the treatment is nonrelativistic; and (d) quantum effects are not taken into account. Indeed, the correct quantum-mechanical calculation for Compton scattering yields the so called Klein-Nishina formula

\[
\frac{d\sigma}{d\Omega} = n_0^2 \frac{1 + \cos^2 \theta}{2} \frac{1}{[1 + \gamma(1 - \cos \theta)]^2}
\times \left[ 1 + \frac{\gamma^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \gamma(1 - \cos \theta)]^2} \right]
\]

(3.11)

where \(n_0\) and \(\theta\) were defined previously, and \(\gamma = \hbar \nu / mc^2\). The cross section has been averaged over all incoming polarizations. By integrating Eq. 3.11, the total cross section can be had. We will not give the complete result here, but the asymptotic expressions have already been presented in Chapter 5, Eq. 2.21.

A comparison of the Thomson (Eq. 3.9) and Klein-Nishina cross sections, including the results obtained in this laboratory for \(\gamma = 1.29\), is shown in Fig. 6.14. We remark that although the Thomson cross section is symmetric about 90°, the Klein–Nishina cross section is peaked forward strongly as \(\gamma\) increases. This is due to a great extent to kinematical factors associated with the Lorentz transformation from the center of mass to the laboratory; note that the center of mass velocity of the (quantum + free electron) system is

\[v = \beta c = c\gamma/(1 + \gamma)\]

where as before \(\gamma = \hbar \nu / mc^2\).

The experimental data are in perfect agreement with the results of Eqs. 3.6 and 3.11, which are among the most impressive and convincing successes of quantum theory. In the following two sections we will describe the experimental verification of these predictions.

### 3.2 The Compton Scattering Experiment

As with any scattering experiment, the apparatus will consist of:

(a) The beam of incident particles, in this case photons.
(b) The target (containing the electrons from which the photons scatter).
(c) The detector of the scattered photons.

3. Compton Scattering

The beam of photons is obtained by collimating the gamma radiation from a Cs\textsuperscript{137} source. As we know (Table 4.2) Cs\textsuperscript{137} (Ba\textsuperscript{137}) emits a gamma ray of energy 0.662 MeV, and the detection techniques have been discussed in Chapter 5. In Fig. 5.28 is shown the pulse-height spectrum of the gamma radiation from Cs\textsuperscript{137}, as obtained with standard equipment; the same detection equipment is used in this experiment with the only difference that the scintillation crystal is placed in a heavy shield.

A schematic of the apparatus is shown in Fig. 6.10. The lead pig \(A\) is fixed and holds the source, which can be introduced through the vertical hole (\(v\)). Another lead shield \(B\) contains the detector and can be rotated about the center, where the target is located. The lead assemblies are rather heavy (approximately 200 lb) and some provisions must be taken for adequate mounting.

For the source, a 35-mCi Cs\textsuperscript{137} sample was used, which was properly encapsulated before being shipped to the laboratory. It should always be transported in a lead container, and when transferred into the lead pig \(A\), it must be handled only by the attached string. The source holder (\(A\)) has a collimator (\(h\)) drilled horizontally, subtending a solid angle of the order of 0.03 sr. Of interest to us will be the density of the photon beam at the target, and the expected value is

\[
\frac{3.7 \times 10^{10} \times 0.035}{4\pi r^2} = 8.8 \times 10^4 \text{ photons/cm}^2\text{-sec}
\]

Where we use \(r = (13.5 \times 2.54) = 34.3\) cm, as read off Fig. 6.10; indeed the observed density of \(41 \times 10^3\) photon/cm\(^2\)-sec is of the predicted order of magnitude.

In contrast to the situation in Rutherford scattering, there is no need to enclose the beam and detector in vacuum or to use a very thin target. We know that gamma rays do not gradually lose energy when traversing matter as a charged particle does, but their interaction can be characterized by a mean free path. For the Cs\textsuperscript{137} gamma ray we find from Fig. 5.34

\[
\lambda = 4.7 \text{ cm in Al}; \quad \lambda = 0.92 \text{ in Pb}
\]

this corresponds to \(10^4\) cm of air, so that the interaction of the photon beam in the air of the apparatus (approximately 100 cm) is indeed negligible. Also, the target thickness can safely be a fraction of a mean free path before the probability for multiple interactions becomes considerable. Aluminum targets \(\frac{1}{2}\)-in. thick are quite adequate for this experiment.

Some special mention must be made of the geometrical shape of the target. We may use a flat target (such as an aluminum plate), in which event the cross section is obtained by considering the interaction of the total beam with the number of electrons per square centimeter of the
FIG. 6.10 Apparatus used for measuring the Compton scattering of Cs\textsuperscript{137} gamma rays from different targets; note that the detector is movable. (a) Top view. (b) Elevation. (c) Use of a flat target when measuring Compton scattering at large angles. By such placement the scattered photons do not have to traverse very large amounts of the target material.
3. Compton Scattering

target†; alternatively, we may use a target of circular cross section (such as a rod), in which event the cross section is obtained by considering the interaction of the beam density (photons per square centimeter) with the total number of electrons in the target.† When using a plate, it is advisable to rotate it so that it always bisects the angle between beam and detector, since otherwise the scattered photons may have to traverse a very large amount of material before leaving the target (see Fig. 6.10c). In that case, however, the amount of scattering material in the beam path varies as $1/\cos(\theta/2)$, and this correction must be applied to the yield of scattered particles. These effects are obviously eliminated when a target of circular cross section is used. In addition, the scattering point is better defined even if the beam is only poorly collimated. On the other hand, accurate evaluation of the flux density at the target is difficult. The results presented here were obtained by using a ¼-in. aluminum rod as the target.

An interesting refinement of the technique is made by observing the recoil electrons in time coincidence with the scattered photon. However, the kinetic energy of the recoil electron is

$$T_e = E - E' = E \frac{\gamma(1 - \cos \theta')}{1 + \gamma(1 - \cos \theta)}$$

which at its maximum value ($\theta = 180^\circ$) is

$$T \text{ (electron)} = 0.662 \times \left(\frac{2.58}{3.58}\right) = 475 \text{ keV}$$

The range of such an electron in aluminum is only 150 mg/cm$^2$ (see, for example, Feather's rule, Chapter 5, Eq. 2.13), which corresponds to approximately 0.06 cm. Thus, the recoil electrons will, in almost all cases, stop in the target. On the other hand, if a plastic scintillator is used as the target, and is viewed with a photomultiplier, the recoil electrons do produce a signal that can be easily detected.

As mentioned before, the detection system consisted of a NaI crystal mounted on a 6655 RCA photomultiplier. The dimensions of the crystal were 1-in. diameter and 1-in. thick. In Fig. 6.11 are shown two typical pulse-height spectra: one at 0° which obviously is dominated by the primary beam and does not change whether the target is in place or removed; and one at 120°, where the photopeak is again clearly observable but appears at a much lower discriminator setting.

By measuring the pulse-height distribution at various angles, we obtain the energy of the scattered photons as it is given by the position of the photopeak, and under the assumption that the discriminator is linear (see Fig. 5.31). To obtain the yield of scattered photons, we may either (a) accept all counts above noise level and apply a correction for the efficiency

† See Fig. 5.1.
of the crystal, or (b) integrate the counts in the photopeak only and apply a correction for the "photofraction" as well as for crystal efficiency. These corrections depend on the crystal size and on the photon energy (which varies with angle); Fig. 6.12 gives the efficiency against energy for a 1-in. diameter, 1-in. thick NaI crystal.† In the present experiment we have chosen the former method, that is, of accepting all counts above noise level.

In this experiment, unlike the Rutherford scattering experiment the background is quite low. When the detector is moved outside the primary beam, which has an angular width of \( \theta_0 \approx 8.5^\circ \), the background counts are less than 0.01 of the scattered counts. Further, the cross section for Compton scattering falls off much more slowly than \( 1/(\sin^4 \theta/2) \), and therefore a higher background can be tolerated; thus useful data can be obtained at large angles. We also note that Cs\(^{137} \) emits mainly the 0.662-MeV gamma ray and 0.514-MeV electrons; these electrons, however, can hardly penetrate the target, much less reach the detector.

### 3.3 Results and Discussion

The results presented below were obtained by students‡ using the apparatus described in the previous section.

---

† From *Scintillation Phosphors*, Harshaw Chemical Company, Cleveland, Ohio.
‡ D. Kohler and A. Rosen, class of 1962.
3. Compton Scattering

At first the beam is surveyed by measuring the profile (see Section 2.3 on Rutherford scattering) and it is ascertained that the pulse-height spectrum has the correct shape, as shown in Fig. 6.11. The beam is found to have an angular spread† of \( \theta_0 = \pm 8.5 \), consistent with the dimensions of the source collimator; from Fig. 6.10 we find \( \tan \theta_0 = 1.5/13.5 = 0.110 \), thus \( \theta_0 \approx 7^\circ \). The peak beam-rate, which is fairly flat near the center is \( 25.4 \times 10^3 \) counts/sec, which corresponds to a density at the detector of

\[
\text{flux} = 5 \times 10^3 \text{ counts/cm}^2\text{-sec}
\]

We have to correct this rate for the efficiency of the crystal, which at this energy (Fig. 6.12) is \( \varepsilon = 0.47 \), and to extrapolate from the density at the detector to the flux density at the target. Assuming a \( 1/r^2 \) dependence and since \( r \) (source detector) = \( 2r' \) (source-target) as shown in Fig. 6.10, we obtain

\[
\text{flux (at target)} = \frac{5 \times 10^3}{\varepsilon} \times \left( \frac{r}{r'} \right)^2 = 42.6 \times 10^3 \text{ photons/cm}^2\text{-sec}
\]

Next, a pulse-height spectrum is taken at different angles and the position of the photopeak carefully measured. Two such spectra have already been shown in Fig. 6.11; these were taken with a discriminator window 0.2 V wide (the abscissa is calibrated in volts). To avoid drifts of the detection system, after each pulse-height spectrum is taken, the detector as-

---

![Figure 6.12: Efficiency of a NaI crystal for the detection of gamma rays, as a function of their energy.](image)

† Full width at half maximum, 17°.
TABLE 6.3
PHOTOPEAK POSITION AS A FUNCTION OF ANGLE

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Discriminator channel</th>
<th>Energy</th>
<th>((1 - \cos \theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.95</td>
<td>0.662</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>7.45</td>
<td>0.620</td>
<td>0.06</td>
</tr>
<tr>
<td>50</td>
<td>5.45</td>
<td>0.455</td>
<td>0.36</td>
</tr>
<tr>
<td>60</td>
<td>4.80</td>
<td>0.400</td>
<td>0.50</td>
</tr>
<tr>
<td>70</td>
<td>4.30</td>
<td>0.358</td>
<td>0.66</td>
</tr>
<tr>
<td>90</td>
<td>3.45</td>
<td>0.287</td>
<td>1.00</td>
</tr>
<tr>
<td>120</td>
<td>2.60</td>
<td>0.218</td>
<td>1.50</td>
</tr>
</tbody>
</table>

The assembly is returned to 0° and the center of the photopeak is scanned. The photopeak values are summarized in the following Table 6.3 and are also shown in Fig. 6.13. We plot the inverse of the photon energy, \(1/E'\), against \((1 - \cos \theta)\).
3. Compton Scattering

TABLE 6.4
DIFFERENTIAL CROSS SECTION FOR COMPTON SCATTERING

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>$E$ (MeV)</th>
<th>Yield (counts/sec)</th>
<th>Efficiency</th>
<th>Corrected $\frac{d\sigma}{d\Omega}$ (experimental)</th>
<th>Klein-Nishina $\times 10^{-28}$ cm$^2$</th>
<th>Nishina $\times 10^{-28}$ cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.662</td>
<td>52.2</td>
<td>0.47</td>
<td>111.0</td>
<td>6.06</td>
<td>5.39</td>
</tr>
<tr>
<td>20</td>
<td>0.614</td>
<td>52.2</td>
<td>0.47</td>
<td>111.0</td>
<td>6.06</td>
<td>5.39</td>
</tr>
<tr>
<td>30</td>
<td>0.564</td>
<td>35.1</td>
<td>0.49</td>
<td>71.5</td>
<td>3.90</td>
<td>4.29</td>
</tr>
<tr>
<td>40</td>
<td>0.503</td>
<td>27.5</td>
<td>0.52</td>
<td>51.0</td>
<td>2.78</td>
<td>3.19</td>
</tr>
<tr>
<td>50</td>
<td>0.452</td>
<td>20.7</td>
<td>0.55</td>
<td>37.7</td>
<td>2.06</td>
<td>2.42</td>
</tr>
<tr>
<td>60</td>
<td>0.402</td>
<td>17.7</td>
<td>0.60</td>
<td>29.5</td>
<td>1.61</td>
<td>1.85</td>
</tr>
<tr>
<td>70</td>
<td>0.353</td>
<td>15.3</td>
<td>0.65</td>
<td>23.5</td>
<td>1.28</td>
<td>1.50</td>
</tr>
<tr>
<td>80</td>
<td>0.320</td>
<td>13.7</td>
<td>0.69</td>
<td>20.0</td>
<td>1.09</td>
<td>1.23</td>
</tr>
<tr>
<td>90</td>
<td>0.289</td>
<td>13.6</td>
<td>0.72</td>
<td>19.0</td>
<td>1.04</td>
<td>1.09</td>
</tr>
<tr>
<td>100</td>
<td>0.263</td>
<td>12.5</td>
<td>0.75</td>
<td>16.6</td>
<td>0.91</td>
<td>1.02</td>
</tr>
<tr>
<td>110</td>
<td>0.242</td>
<td>13.4</td>
<td>0.78</td>
<td>17.2</td>
<td>0.94</td>
<td>1.01</td>
</tr>
<tr>
<td>120</td>
<td>0.225</td>
<td>14.8</td>
<td>0.81</td>
<td>18.4</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td>130</td>
<td>0.211</td>
<td>14.8</td>
<td>0.84</td>
<td>17.6</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>140</td>
<td>0.201</td>
<td>16.6</td>
<td>0.86</td>
<td>19.3</td>
<td>1.05</td>
<td>0.99</td>
</tr>
<tr>
<td>150</td>
<td>0.193</td>
<td>17.1</td>
<td>0.88</td>
<td>19.4</td>
<td>1.06</td>
<td>0.98</td>
</tr>
</tbody>
</table>

$(1 - \cos \theta)$; according to Eq. 3.6, a straight line should be obtained, since

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{mc^2} (1 - \cos \theta)$$

This is indeed the result, and the slope of the line gives $1/mc^2$; from a least-squares fit we obtain

$$mc^2 = 505 \pm 12 \text{ keV}$$

in very good agreement with the known value of the electron mass.

We thus conclude that Eq. 3.6 is very well verified and that our explanation of the Compton frequency shift is firmly supported by these data.

We next turn to the evaluation of the differential cross section. As explained before, all counts above the 1 V level were accepted, and data were taken at several angles. The results are summarized in Table 6.4: here column 1 gives the angle and column 2 the corresponding photon energy; column 3 gives the raw yield in counts per second at that particular angle; column 4 gives the efficiency of the detector for the photon energy (of column 2) as obtained from Fig. 6.12, and the corrected counting rate appears in column 5.
To obtain the cross section we note that

\[
\frac{d\sigma}{d\Omega} = \frac{\text{yield}}{(d\Omega)Nl_0}
\]

From Fig. 6.10 we see that the detector solid angle is given by

\[
d\Omega = \frac{\text{crystal area}}{\pi r^2} = \frac{5.07 \text{ cm}^2}{(34.3)^2 \text{ cm}^2} = 4.3 \times 10^{-2} \text{ sr}
\]

For the total number of electrons in the target, we have

\[
N = \pi \left(\frac{d}{2}\right)^2 h \rho \frac{N_0}{A} Z
\]

Where

- \(d = \text{diameter of target} = 1.27 \text{ cm}\)
- \(h = \text{height of target} = 10 \pm 2 \text{ cm}\)
- \(\rho = \text{density of aluminum} = 2.7 \text{ gm/cm}^3\)
- \(N_0 = \text{Avogadro’s number} = 6 \times 10^{23}\)
- \(A = \text{atomic weight of aluminum} = 27\)
- \(Z = \text{atomic number of aluminum} = 13\)

thus

\[N = 10^{25} \text{ electrons}\]

For \(I_0\), the flux density at the target, we use the previously obtained value

\[I_0 = 42.6 \times 10^3 \text{ photons/cm}^2\text{-sec}\]

so that finally

\[
\frac{d\sigma}{d\Omega} = \frac{\text{corrected yield}}{4.3 \times 10^{-2} \times 10^{25} \times 42.6 \times 10^3} = \frac{\text{corrected yield}}{1.83 \times 10^{27}}
\]

The values of the differential cross section obtained in this fashion are given in column 6 of Table 6.4, and are also plotted in Fig. 6.14. The solid line in Fig. 6.14 gives the theoretical values for \(d\sigma/d\Omega\) derived from the Klein–Nishina formula (Eq. 3.11) for \(\gamma = 1.29\), while the dashed curve represents the Thomson cross section.

The agreement of the angular dependence of the experimental points with the theoretical curve is indeed quite good and clearly indicates the inadequacy of the Thomson cross section for the description of the scatter-

\[\uparrow\text{This is obtained by estimating the length of target intercepted by the beam. We assume that the angular spread in the vertical direction is the same as in the horizontal, } \theta_0 = \pm 8.5\text{ and use } r = 34.3 \text{ cm; see also Fig. 6.10.}\]
4. Mössbauer Effect

4.1 General Considerations

In the two experiments previously described, we could visualize the scattering process as if it were a collision of two billiard balls in which the incoming alpha particle or photon maintained its identity but suffered a change in momentum and energy. The phenomenon of scattering can, however, also be visualized as the absorption by the target of quanta of the incoming beam, with the subsequent re-emission of these quanta; this was the model we used in the derivation of the Thomson scattering cross section in the previous chapter.

![Graph](image-url)

**Fig. 6.14** The results obtained for the scattering cross section of Cs$^{137}$ gamma rays as a function of angle. The solid line is the prediction of the Klein-Nishina formula for that particular energy; the dotted line is the Thomson cross section.
passage through resonance, which gives rise to "transient effects"; by fast is meant fast as compared to the characteristic relaxation times of the system.

Indeed, from Fig. 8.14b we estimate the width of the line to be $\Delta H \approx 0.4$ gauss full width at half maximum (fwhm), and using Eq. 3.17† we obtain

$$T_2 = \frac{1}{2\Delta \nu} = \frac{\pi}{\Delta \omega} = \frac{\pi}{\gamma \Delta H} = \frac{3.14}{2.67 \times 10^4 \times 0.4} \sim 3 \times 10^{-4} \text{ sec}$$

(4.7)

which is comparable with the time taken by the field to sweep over the resonance as shown by Eq. 4.6; thus the spin-spin interaction hardly has the time to restore equilibrium conditions during the short time interval‡ that the sample finds itself in resonance with the radiofrequency.

The explanation for the "wiggles" of Fig. 8.14a and Fig. 8.16 is the following. The spins all precess about the $z$ axis at a frequency $\omega$, which slowly varies with the sweeping field as

$$\omega = \gamma (H_0 + H_{\text{sweep}})$$

(4.8)

However, the spins are not in phase; that is, their azimuthal angle $\phi$ is random. When the field reaches an $H$ value such that the precession frequency $\omega$ equals the applied radiofrequency $\omega_0$, resonance occurs and all the spins are brought into phase (that is, they all cluster in the azimuthal plane of the rotating radiofrequency field $H_1$). As the constant field increases beyond the resonance value, the spins continue to precess at the frequency $\omega$, which again differs from $\omega_0$, but they remain in phase for a time interval of the order of $T_2$. However, such coherently precessing spins will induce in the radiofrequency coil a voltage at the frequency $\omega$; the two close frequencies $\omega$ of the precessing spins and the applied $\omega_0$ which are not exactly equal will interfere (beat) as their relative phase angle

† For simplicity we have assumed $g(\nu)$ to have a triangular shape (see Eq. 2.12). Using

$$\int g(\nu) \, d\nu = 1$$

we have

$$1 = \frac{1}{2} g(\nu_0)(2\Delta \nu).$$

Similarly, for a Gaussian we obtain

$$T_2 = \frac{1}{2} g(\nu_0) = \frac{2}{\gamma \Delta H}.$$

‡ The reader may recognize that this is only a case of a generalized uncertainty principle in experimental procedure. With a detector of band width $\Delta f$, the signal-to-noise ratio improves with decreasing $\Delta f$ as $S/N \propto 1/\sqrt{\Delta f}$; however, for a given $\Delta f$, we must wait at least a time interval $\Delta t \sim 1/\Delta f$ in order to make even an approximate measurement, and for an accurate value several time constants $\Delta t$ need to have elapsed.
changes; this results in the "wiggles" we observe. They do decay exponentially as the phase coherence of the precessing spins becomes destroyed with a time constant $T_2$.

If a linear sweep is assumed, the beat signal has the form

$$e^{-t/T_2} \cos \left[ \frac{1}{2} \gamma \frac{dH}{dt} t^2 \right]$$

where $t = 0$ when resonance is traversed. We also note that the beat frequency increases with time since

$$\omega_b = \frac{1}{2} \gamma \frac{dH}{dt} t$$

Thus, from a measurement of the wiggle pattern, information about $T_2$ can be obtained.

When $T_2$ is long, and sufficient radiofrequency power is applied, it is possible that the phase coherence may not have completely decayed before the magnetic field value $H$ again approaches resonance; in this case wiggles are also observed before resonance as shown in Fig. 8.15a, taken with a linear time sweep of 0.2 msec/cm; Fig. 8.16 is a semilog plot of the amplitude of the wiggles (after resonance) of Fig. 8.15a against time. We see that the data fit an exponential, and yield

$$T_2 \approx 5 \times 10^{-4} \text{ sec}$$

of the same order as given by Eq. 4.7.

Fig. 8.15 Nuclear magnetic resonance signals for protons when the passage through resonance is very rapid. The sample is water saturated with lithium fluoride. (a) The oscilloscope sweep is linear; note that passage through resonance occurs only once. (b) The oscilloscope sweep is sinusoidal and passage through resonance occurs twice.
Nuclear Magnetic Resonance of Protons

\[ T_2 = 4.8 \times 10^{-4} \text{ sec} \]

FIG. 8.16 Semilog plot of the amplitude of the "wiggles" of the resonance signal shown in Fig. 8.15 plotted against time. Note that it yields an exponential decay of the amplitude with a characteristic time constant of $4.8 \times 10^{-4} \text{ sec}$.

Just as the inhomogeneity of the magnetic field will broaden the line, however, so it will also destroy the phase coherence (since different parts of the sample precess at different frequencies $\omega$). In fact, a detailed analysis\(^\dagger\) shows that the pattern of Fig. 8.15a is typical of an inhomogeneous field. Thus both values for the effective $T_2$ obtained in Eqs. 4.8 and 4.12 are to be interpreted as due to field inhomogeneities; the true $T_2$ time for these samples is longer.

In concluding, we mention Fig. 8.15b, which is the same as 8.15a but shows the resonance signal when a sinusoidal sweep is applied to the oscilloscope $x$ axis. Therefore resonance appears twice on the oscilloscope, during both increasing and decreasing field; also now the horizontal axis is linear in field rather than in time. The traces shown in Figs. 8.15a and 8.15b were obtained by students\(^\ddagger\) using a saturated solution of lithium fluoride (LiF) in water. The sweeping field was at 60 cps and had a maximum value of the order of 10 gauss.

\(^\dagger\) E. R. Andrew, loc. cit., p. 134.
\(^\ddagger\) D. Boyd and P. Nichols, class of 1963.
4.4 Transistorized Nuclear Magnetic Resonance Detector

As a final example we present in Fig. 8.17 a simple transistor circuit† which can be used for the observation of proton nuclear magnetic resonance. Transistor $T_1$ (2N502) is the oscillator, feedback being provided by the 10-$\mu$F capacitor from the collector to the emitter. The frequency is determined by the $LC$ circuit formed by the radiofrequency (the sample) and the variable condenser; it is possible to vary the frequency continuously from 2 to 80 Mc/sec by using coils of the appropriate inductance for each of four ranges. Diode $D_1$ (1N56) rectifies the radiofrequency and $T_2$ (2N247) amplifies the audio signal. The output is fed directly to an oscilloscope. The radiofrequency level is adjusted by varying the 10-K potentiometer in the emitter of $T_1$.

As before, the sample is 1 cm$^3$ of water that is doped with manganese nitrate [Mn(NO$_3$)$_2$] and is placed inside the radiofrequency coil; the field is modulated at 60 cps. The important difference between this and the previous circuit is that transistors are constant current sources. Thus out of the collector of $T_1$ flows a current whose amplitude depends on the base-emitter bias, and the radiofrequency voltage at point $A$ is entirely determined by the impedance of the $LC$ circuit. Any change in the $Q$ of the coil appears immediately as a change in the radiofrequency level at point $A$ (or $B$). This is the fact that accounts for the great simplicity of the circuit of Fig. 8.17.

4. Nuclear Magnetic Resonance of Protons

Fig. 8.18 Nuclear magnetic resonance signals from protons obtained with the circuit shown in Fig. 8.17 as a function of the amplitude of the radiofrequency. Note that initially the output signal increases with increasing radiofrequency but at a level of approximately 0.5 V the sample is saturated and the signal begins to decrease.

Fig. 8.19 Proton nuclear magnetic resonance signal obtained with the circuit shown in Fig. 8.17 and displayed on an expanded time scale. The magnetic field is of the order of 8 kilogauss.
Data obtained by a student† using this circuit at a frequency of 33,830 Mc/sec are shown in Fig. 8.18 for various levels of radiofrequency voltage across the coil. The magnet was swept at 60 cps, and the oscilloscope sweep is linear at $2 \times 10^{-3}$ sec/cm; the vertical scale is 200 mV/cm for all but trace (a) where the gain was turned up by a factor of 10. We note that as the level of the radiofrequency is increased, the output at the collector of $T_2$ increases also; however, for radiofrequency levels beyond $\approx 0.5$ V the output level begins to decrease as shown in Fig. 8.18; it almost disappears at $\approx 0.8$ V. This is due to the saturation of the sample with increasing $H_1$ as discussed before and predicted by Eq. 3.14. Finally, in Fig. 8.19 the resonance signal for 0.6 V of radiofrequency and with an expanded horizontal scale of $0.4 \times 10^{-3}$ sec/cm is shown, indicating clearly the “wiggles” after passage through resonance.

5. Electron Paramagnetic Resonance

5.1 General Considerations

As noted in Section 1, when radiofrequency resonance is established between electronic rather than nuclear energy levels (always in an external magnetic field), we speak of electron paramagnetic resonance (epr). We know, however, that an energy level is split into sublevels only if its angular momentum $J$ is different from zero; this condition, for example, is met by the free electron where the total angular momentum $J$ is just its intrinsic angular momentum, the spin $S = \frac{1}{2}$.

Similarly, from our study of optical spectroscopy in Chapter 7, we know that in many atoms the ground state may have $J \neq 0$ (also many excited states with $J \neq 0$ exist) so that paramagnetic resonance can be established between the Zeeman sublevels of such an atomic state.‡ If we turn to solids, however, it is much more difficult to find electronic states with $J \neq 0$: this is due to the fact that in the chemical binding of atoms into molecules, the valence electrons get paired off, so that each atom appears to have a completely closed shell. For example, in NaCl, the sodium has a $^2S_{1/2}$ electron ($n = 3, l = 0$) outside closed shells, and the chlorine has a $^2P_{3/2}$ electron hole ($n = 3, l = 1$) inside closed shells. However, in the NaCl molecule, the sodium appears as a Na$^+$ ion, and hence presents a closed shell configuration, whereas the chlorine appears as a Cl$^-$ ion again with completely closed shells. Consequently, the NaCl molecule is completely diamagnetic.

Nevertheless, it is known from the work on static magnetic susceptibilities (especially at low temperatures) that certain salts show strong

† J. S. Weaver, class of 1962.
‡ In that instance, however, it is not usually called electron paramagnetic resonance.
paramagnetism (contain ions with permanent magnetic moments of the order of $\mu_0$). In particular, compounds containing ions of the “transition elements” of the periodic table are frequently found to be paramagnetic. As an example we mention copper sulfate, $[\text{Cu}(\text{SO}_4)]$, in which compound the double valence results in a $\text{Cu}^{2+}$ ion. For copper the $n = 1, 2, \text{ and } 3$ shells are completely filled and it has one electron in the $4s$ state, so that $\text{Cu}^{2+}$ has a hole in the $3d$ shell; thus the ground state of the $\text{Cu}^{2+}$ ion has $l = 2$, $s = \frac{1}{2}$, and, consequently, $J \neq 0$, so that it does possess a magnetic dipole moment. In an external magnetic field, the ground state will be split into sublevels and resonance between them can be established and is indeed observed. The actual situation, however, is more complicated due to the electric field of the crystalline lattice; this will be discussed further in Section 5.3.

Electronic moments can also be found in solids when the chemical bond is broken, as in organic free radicals. Especially, the organic salt DPPH, diphenyl-pieryl-hydrazil, $(C_6H_{14})_2N-NC_6H_5(NO_2)_3$ shows a very strong and narrow resonance line, with a $g$ factor very close to 2.00 (the free electron value) and is therefore frequently used as a standard. The structure of the molecule is shown in Fig. 8.20, and the “free-electron” behavior comes from the single electron bond in one of the nitrogens.

Paramagnetic resonance has also been observed in other materials where unpaired electrons may exist, as in crystals with lattice defects, in ferromagnetic materials, and in metals and semiconductors.

Since the energy splitting in paramagnetic resonance involves the Bohr magneton rather than the nuclear magneton, the frequencies will be 2000 times larger than the nuclear magnetic resonance frequencies in the same magnetic field. Thus electron paramagnetic resonance can be observed at megacycle frequencies in fields of a few gauss, or otherwise in the microwave region in a magnetic field of a few kilogauss. The latter alternative is almost always chosen, since it presents distinct advantages.

![Chemical structure of DPPH](image)

Fig. 8.20 Chemical structure of DPPH (diphenyl-pieryl-hydrazil), $(C_6H_{14})_2N-NC_6H_5(NO_2)_3$. 
(a) For each transition the absorbed energy is much larger, and thus the signal-to-noise ratio is much improved, as also discussed in Section 4.1.
(b) A high magnetic field is used, thus providing separation between levels that are intrinsically wide and would remain partially overlapped at low fields.

The resonance condition is detected, as in the case of nuclear magnetic resonance by the absorption of energy, and for this reason solids and liquids are much easier to study than gases with their very low densities. Much of our previous discussion on transition probabilities and relaxation mechanisms is equally applicable to electron paramagnetic resonance. Again we depend on the Boltzmann distribution for the creation of differences in the population of the various sublevels; these differences are much larger than in nuclear magnetic resonance, since the energy intervals involved are 2000 times larger (see Eq. 1.2 and Eq. 2.2).

A difficulty with electron paramagnetic resonance, however, is that the width of the resonance lines may be prohibitively large, since both the spin-lattice and spin-spin interactions are stronger than in the nuclear magnetic resonance case (see Eq. 3.16). In order to reduce the line width, the sample may be cooled to low temperatures, which lengthens the spin-lattice relaxation time. To decrease the spin-spin interaction, on the other hand, the paramagnetic ions are diluted in a diamagnetic salt, which effectively increases the distance between the spins (again see Eq. 3.16).

As compared to atomic spectroscopy, where the Zeeman splitting of the sublevels was only a small perturbation on the optical transition, here we measure directly the spacing between these sublevels. However, in optical spectroscopy the atom could be excited to one of any of its higher levels; in electron paramagnetic resonance we are mainly working with the ground state of the ion, and some of the levels closest to it.

When measuring electron paramagnetic resonance lines in solids, a great variety of \( g \) factors is obtained. This is due to the differences in the coupling of the unpaired electron's spin with the orbital angular momentum; the strength of this coupling depends very much on the position (in energy) of the adjacent levels of the ion as they are modified by the crystalline field. Further, the electron paramagnetic resonance lines show hyperfine structure characteristic of the interaction of the nucleus with the ionic energy levels; this structure in turn can be used to positively identify small traces of an element contained in some unknown sample.

Similarly, the organic free radicals show characteristic lines (\( g \) factors) which can be used to identify them and show hyperfine structure as well.

\[ \dagger \text{This is possible because of the strong excitation provided by an arc source, or high-voltage discharge, etc.} \]
In fact, a radical that has no structure (like the DPPH) may exhibit such effects when the sample is prepared in a liquid solution.

It is for these reasons that electron paramagnetic resonance has become of great value in the understanding of crystalline materials as well as of organic structures; the study of these effects has by now so increased that they form independent fields of research, which we will not be able to discuss. In Section 5.2 we will describe the experimental apparatus and show the line observed from DPPH, while in Section 5.3 we will briefly discuss the Cu$^{2+}$ ion and the results obtained from Cu(SO$_4$)·7H$_2$O, and MnCl$_2$·4H$_2$O.

5.2 ELECTRON PARAMAGNETIC RESONANCE APPARATUS

In this laboratory electron paramagnetic resonance was observed at a microwave frequency of the order of 10,000 Mc ($\lambda_f \approx 3$ cm). A schematic of the interconnections of the equipment is shown in Fig. 8.21, and Figs. 8.23 and 8.25 give some of the details of the microwave plumbing.

The system consists, basically, of the $X$-band Klystron oscillator and plumbing to transport the power to a microwave cavity in which the sample is placed. The sample, in turn, must be subjected to a permanent field $H_0$, which is usually continuously varied with a motorized drive. A microwave bridge is used to balance out the power and give a signal only when resonant absorption occurs; this signal is fed to a Keithley electrometer whose output in turn drives the $y$ axis of a recorder. The $x$ axis is driven in proportion to the slow variation of $H_0$. For very weak signals a phase-sensitive ("lock-in")

![Fig. 8.21 Schematic arrangement of apparatus for an electron paramagnetic resonance experiment in the microwave region.](image-url)
detector may be used, in which case modulation of the output signal must be provided; either the Klystron may be modulated in an “on-off” mode or the magnetic field may be modulated with sweep coils.

Let us now examine the various elements of the apparatus in slightly more detail†.

(1) The microwave oscillator. This is a 723A reflex Klystron‡ mounted in a TS/13-AP power supply; a sketch of the Klystron is given in Fig. 8.22, and in Fig. 8.23 some of the plumbing and auxiliary equipment included in the power supply are shown.

The frequency can be tuned over a small range either by a mechanical adjustment or by varying the reflector voltage. The tube has a few “modes” of operation but most power is achieved at a reflector voltage of $-400$ V with the grids at ground potential. Figure 8.24 shows the effect of reflector voltage; it can be seen how the superposition of a square-wave voltage on the reflector may modulate either the frequency or amplitude of the tube.

† We cannot discuss microwave techniques here, but the reader should consult the well-known M.I.T. Radiation Laboratory Series, McGraw-Hill, 1947; also A. Bronwell and R. Beam, Theory and Application of Microwave Techniques, McGraw-Hill, 1947.

‡ Both the Klystron and power supply described here are war surplus equipment, but now many improved Klystrons are available commercially.
5. Electron Paramagnetic Resonance


FIG. 8.24 The power output and frequency shift of a Klystron oscillator in its various modes, as a function of reflector voltage.
Fig. 8.25 The microwave plumbing used for the electron paramagnetic resonance experiment. The branches (B) and (C) form the two arms of a bridge. (1) TS 13/AP microwave generator, (2) Slotted line, (3) Rigid twist, (4) Magic tee, (5) Detector, (6) Attenuator, (7) Phase shifter, (8) Tuning stub or matched load, (9) Equivalent of tuned cavity, (10) Tuning stub, (11) Sample, (12) Electromagnet pole faces, (13) Bridge output.
The wave guide system. This is shown in Fig. 8.25. It consists of a transport section (A) and two sections (B) and (C) which form the arms of a bridge; (B) contains the sample while (C) contains a dummy load. To balance the bridge we adjust the attenuator (6) (for resistive balance), the phase shifter (7) (for reactive balance), and the shorting plunger (8).†

At this point we inquire into the configurations of the field inside the rectangular wave guide. We know that only certain modes will propagate‡ (without attenuation) and the wavelength in the guide \( \lambda \) is given by

\[
\frac{1}{\lambda^2} = \frac{1}{\lambda_f^2} - \frac{1}{\lambda_e^2} = \frac{1}{\lambda_f^2} - \frac{(m/a)^2 + (n/b)^2}{4}
\]

(5.1)

where \( \lambda_f \) is the free space wavelength and \( a \) and \( b \) are the inner dimensions of the guide; \( m \) and \( n \) are integers. Since

\[
a = 2.29 \text{ cm}, \quad b = 1.02 \text{ cm}, \quad \text{and} \quad \lambda_f \approx 3.2 \text{ cm}
\]

we find that only the \( m = 1, n = 0 \) mode can propagate, and

\[
\lambda_e = 4.5 \text{ cm}
\]

In this mode, the electric field is completely transverse to the axis of the guide; this is called the \( TE_{10} \) mode. The field lines for the traveling \( TE_{10} \) wave are shown in Fig. 8.26, where the density of field lines is proportional to the field strength.

The microwave cavity and sample. The cavity is the analogue of the radiofrequency coil in the nuclear resonance apparatus. In the apparatus shown in Fig. 8.25, however, no special cavity was used. Instead a part of the wave guide (9) was ended with a shorting stub (10) and as a result of this, power is reflected and a standing wave is set up. We first adjust the sample to be in the middle of the magnet polefaces, and then adjust the sliding stub (10) so that maximum \( H \) field exists at the sample. From the configuration of the standing wave pattern (which is different from Fig. 8.26) it is found that maximum \( H \) field occurs at a distance \( x \) from the short

\[
x = \left(\frac{1}{2} + \frac{p}{2}\right) \lambda_e
\]

with \( p \) an integer. We also note that since the microwave field must be normal to \( H_0 \) it is preferable to place the guide in the magnet with its wide side parallel to the polefaces.

† Alternatively, a matched load may be used to terminate the dummy arm of the bridge.‡ See, for example, J. D. Jackson, *Classical Electrodynamics*, John Wiley, Chapter VIII.
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(4) The magic T. This is the heart of the bridge circuit and is shown in Fig. 8.27. It has the property that microwave power flowing in from (A) can branch into (B) and (C) but not into (D), which is the direction normal to the plane of (A), (B), and (C); on the other hand, power returning (that is, reflected) from (B) or (C) can flow into (D) and is split equally between (A) and (D). This is due to the different polarization of the electromagnetic field that can propagate in the four elements of the magic T, and to the configuration of the slots leading to the four arms. When the T is used in a bridge circuit as in Fig. 8.25, power reflected from (B) and (C) are superimposed at (D) to give null output.

(5) The detector. This is connected to arm (D) of the magic T and consists of a microwave diode rectifier. Power is fed in from a small probe protruding into the wave guide, and the output is fed to a Keithley electrometer† which is operated as a millivoltmeter. The Keithley output is then fed to a pen recorder as noted earlier.

(6) Auxiliary equipment. This is used for tuning and monitoring the microwave system. For example, (2) in Fig. 8.25 is a slotted line that is

† See Chapter 4, Section 3.4.
used to measure the $E$ field (time average) along the axis of the guide. If a standing wave exists, then the diode connected to the slotted line probe will register maxima and minima (spaced, obviously, $\lambda_g/2$ cm apart). By tuning the system we wish to eliminate such reflections—that is, make the maximum to minimum ratio of the probe readings close to 1.

Also attached to the Klystron output (and incorporated in the TS-13/AP) is an attenuator and a tunable microwave cavity as shown in Fig. 8.23. The cavity affects the frequency and tuning of the Klystron but is mainly intended as a wavemeter. It has a high $Q$, and when it is tuned onto the Klystron frequency, a dip in the output meter appears; if the cavity has been carefully calibrated against a frequency standard, the reading of the adjusting dial can immediately give the Klystron wavelength to good accuracy. The purpose of the attenuator is to reduce the coupling of the Klystron to the load so that the Klystron operation is not much affected by changes in load conditions†. The power supply has a “sync” input which can be used to synchronize the modulation of the reflector voltage with a reference source as is needed when a phase-sensitive detector is used.

(7) The magnet. This must be an electromagnet rather than a permanent magnet, since the frequency range is so limited. It is usually swept continuously by a slow motor driving a rheostat. Fast sweep coils for modulation purposes may be added.

† So-called frequency pulling; a unidirectional device (such as one made out of ferrite) is superior for isolation without loss of transmitted power.
Fig. 8.28 Calibration of the electromagnet used in the electron paramagnetic resonance experiments.

Fig. 8.29 Electron paramagnetic resonance signal from DPPH obtained at a frequency of 10,000 Mc. The abscissa gives the current through the electromagnet in the field of which the sample is located. Note the small width of the line and the good signal-to-noise ratio.