implies that in the process of going from energy state \( n \) to energy state \((n-1)\nu\) the source would emit a burst of electromagnetic energy of energy content \( \nu \).

Einstein assumed that such a burst of emitted energy is initially localized in a small volume of space; and furthermore that it moves away from the source with velocity \( c \), instead of spreading out in the manner characteristic of moving waves. He assumed that the energy content \( e \) of such a bundle, or quantum of energy, is related to its frequency \( \nu \) by the equation

\[
e = h\nu
\]

He also assumed that in the photoelectric process one of the quanta is completely absorbed by an electron in the photocathode. This would impart to the electron an energy equal to \( \nu \). If this energy is greater than the quantity \( \Delta E \), the electron can escape from the photocathode, where \( \Delta E \) is the energy which the electron must expend in reaching the surface of the photocathode plus the energy \( W \) required to overcome the attractive forces which we know must be present at the surface since electrons do not normally escape. The kinetic energy of the electron after escaping from the photocathode will be equal to \( E = \nu - \Delta E \). For an electron originating at the surface, \( \Delta E \) will be just equal to \( W \), and \( E \) will have its maximum value

\[
E_{\text{max}} = \nu - W
\]

where \( W \) is a constant which depends on the composition of the photocathode.

According to this theory, the rate of emission of photoelectrons will be proportional to the flux of quanta incident upon the photocathode, which in turn will be proportional to the intensity of the incident electromagnetic radiation. This is in agreement with Lenard's observation that the photoelectric current is proportional to the intensity. As can be seen from equation (3-8), the theory also agrees with the observation that \( E_{\text{max}} \) does not depend on the intensity. Furthermore, the theory removes the difficulty concerning the time required for the photoelectrons to receive enough energy. This difficulty arose in the wave theory because the energy of the source was spread uniformly over the entire wave front. In the quantum theory the energy is concentrated in the quanta. Within a very short time after the source is turned on, a quantum will strike the photocathode where it will be absorbed by one of the electrons, and the ejection of that electron will result.

Einstein's theory, as summarized by equation (3-8), makes the prediction that the maximum kinetic energy of the photoelectrons should be a linear function of the frequency of the incident electromagnetic radiation. This prediction was tested in 1916 by Millikan, who measured \( E_{\text{max}} \) for radiation in the frequency range 6 to 12 \( \times 10^{14} \) sec\(^{-1}\). The nature of his data is indicated in figure (3-5). The experiment shows that \( E_{\text{max}} \) actually is a linear function of the frequency \( \nu \). According to equation (3-8), the slope of the straight line fitting the data should be equal to Planck's constant \( h \), and its intercept on the \( \nu \) axis should be equal to \( W/h \). Thus the photoelectric experiments, with the aid of Einstein's theory, could be used to determine the numerical value of \( h \). The value of \( h \) which was obtained agreed to within better than 0.5 percent with the value of that constant which had been determined by the completely independent method of fitting Planck's theory to the experimental black body spectrum. This finding represented a great triumph for the quantum theories.

8. The Compton Effect

In 1923 Compton discovered that, when a beam of X-rays of well-defined wavelength \( \lambda_0 \) is scattered through an angle \( \theta \) by sending the radiation through a metallic foil, the scattered radiation contains a component of well-defined wavelength \( \lambda_c \) which is longer than \( \lambda_0 \). This phenomenon is called the Compton effect. As we shall see, it provides extremely convincing evidence for the existence of quanta. Although a description of the production of X-rays and the measurement of their wavelength will not be presented until Chapter 14, it is desirable to discuss the Compton effect at this time.
Taking the idea of a quantum as a localized bundle of energy quite literally, X-rays consist of electromagnetic radiation of wavelength which is shorter than that of visible light by a factor of the order of $10^4$. This is proved by experiments which show that X-rays are diffracted and polarized just like light. In Compton’s experiment the wavelength $\lambda_0$ of the incident radiation was 0.708 × $10^{-8}$ cm. The wavelength $\lambda_1$ was observed to depend on the angle of scattering $\theta$, but not on the material comprising the foil. A typical set of data is shown in figure (3-6) for

$$E = hv$$

(3-9)

Taking the idea of a quantum as a localized bundle of energy quite literally, we shall consider it to be a particle of energy $E$ and momentum $p$. Such a particle must, however, have certain quite specialized properties. Consider equations (1-20) and (1-24), which give the total relativistic energy of a particle in terms of its rest mass $m_0$ and its velocity $v$:

$$E = m_0c^2\sqrt{1 - v^2/c^2}$$

Since the velocity of a quantum equals $c$, and since its energy content $E = hv$ is finite, it is apparent that the rest mass of a quantum must be zero. Thus the quantum can be considered to be a particle of zero rest mass, and of total relativistic energy $E$ which is entirely kinetic. The momentum of a quantum can be evaluated from the general relation (1-25) between the total relativistic energy $E$, momentum $p$, and rest mass $m_0$. This is

$$E^2 = c^2p^2 + (mc^2)^2$$

For a quantum the second term on the right is zero, and we have

$$p = E/c = hv/c = h/\lambda$$

(3-10)

where $\lambda$ is the wavelength of the electromagnetic radiation associated with the quantum. It is quite interesting to note that Maxwell’s classical wave theory of electromagnetic radiation leads to an equation $p = E/c$, where $p$ represents the momentum content per unit volume of the radiation and $E$ represents the energy content per unit volume.

Now the frequency $\nu$ of the scattered radiation was observed to be independent of the material in the foil. This implies that the scattering does not involve entire atoms. Compton assumed that the scattering was due to collisions between the quanta and individual electrons in the foil. He also assumed that the electrons participating in this scattering process are free and initially stationary. Some a priori justification of this assumption can be found from considering the fact that the energy of an X-ray quantum is several orders of magnitude greater than the energy of an ultraviolet quantum, and from our discussion of the photoelectric effect it is apparent that the energy of an ultraviolet quantum is comparable to the energy with which an electron is bound in a metal. Consider, then, a collision between a quantum and a free stationary electron, as in figure (3-7). In the diagram on the left, a quantum of total relativistic energy $E_0$ and momentum $p_0$ is incident on a stationary electron of rest mass energy $mc^2$. In the diagram on the right, the quantum is scattered at an angle $\theta$ and moves off with total relativistic energy $E_1$ and momentum $p_1$, while the electron recoils at an angle $\phi$ with kinetic energy $T$ and momentum $p$. Compton applied the conservation of momentum and total...
relativistic energy to this collision problem. Momentum conservation requires

\[ p_0 = p_1 \cos \theta + p \cos \phi \]  

(3-11)

and

\[ p_1 \sin \theta = p \sin \phi \]  

(3-12)

Squaring these equations, we obtain

\[ (p_0 - p_1 \cos \theta)^2 = p^2 \cos^2 \phi \]  

(3-13)

and

\[ p^2 \sin^2 \theta = p^2 \sin^2 \phi \]  

(3-14)

Adding, we find

\[ p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta = p^2 \]  

(3-15)

Conservation of total relativistic energy requires

\[ E_0 + m_0 c^2 = E_1 + T + m_0 c^2 \]  

(3-16)

Thus

\[ E_0 - E_1 = T \]

According to equation (3-10), this is

\[ c(p_0 - p_1) = T \]  

(3-17)

According to equations (1-22') and (1-25),

\[ (T + m_0 c^2)^2 = c^2 p^2 + (m_0 c^2)^2 \]

(3-18)

so

\[ T^2 + 2Tm_0 c^2 = c^2 p^2 \]

or

\[ T^2/c^2 + 2T = p^2 \]  

(3-19)

Evaluating \( p^2 \) from equation (3-13), and \( T \) from (3-15), we have

\[ (p_0 - p_1)^2 + 2m_0 c(p_0 - p_1) = p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta \]

which reduces to

\[ m_0 c(p_0 - p_1) = p_0 p_1 (1 - \cos \theta) \]

or

\[ \left( \frac{1}{p_0} - \frac{1}{p_1} \right) = \frac{1}{m_0 c} (1 - \cos \theta) \]

Multiplying through by \( h \) and applying equation (3-10), we obtain

\[ \lambda - \lambda_0 = \lambda_c (1 - \cos \theta) \]  

(3-20)

where

\[ \lambda_c \equiv \frac{h}{mc} = 0.02426 \times 10^{-8} \text{ cm} \]

is the so-called Compton wavelength. Equation (3-20) is the Compton equation. It predicts that the increase in wavelength \( \lambda - \lambda_0 \) of the scattered electromagnetic radiation depends only on the scattering angle \( \theta \) and the universal constant \( h/m_0 c \) and is independent of the wavelength of the incident radiation. The equation was verified quantitatively by the experiments of Compton and others. The recoil electrons were observed experimentally by Bothe and by Wilson (1923). Bothe and Geiger (1925) showed that, in the scattering of a single quantum by an electron, the scattered quantum and the recoil electron appear simultaneously. The energy of the recoil electrons were measured by Bless (1927) and found to be in agreement with the predictions of the theory. These experiments, and their interpretation in terms of the theory, provide even stronger evidence for the existence of quanta than does the photoelectric effect.

9. The Dual Nature of Electromagnetic Radiation

Einstein’s quantum theory of the photoelectric effect was very successful in explaining a phenomenon which simply could not be understood on the basis of the theories of classical physics. Nevertheless, many physicists were very reluctant to accept it because it was completely contrary to the well-established wave nature of electromagnetic radiation. After the discovery and interpretation of the Compton effect, the existence of quanta could no longer be questioned. As a result physics was in what seemed at the time to be a very uncomfortable situation. On the one hand, electromagnetic radiation manifests certain properties, such as diffraction, which could only be explained in terms of wave motion. On the other hand, some of its properties, such as those evident in the photoelectric and Compton effects, could only be explained in terms of quanta which, being localized, have essentially the characteristics of particles. Electromagnetic
radiation appeared to possess a split personality, sometimes behaving as if it were composed of waves and sometimes behaving as if it were composed of particles.

The experimental facts concerning the properties of electromagnetic radiation, as well as the interpretations of these properties in terms of the existence of both wave aspects and particle aspects, remain essentially unchanged today. However, as a consequence of the broader point of view provided by the development of the theory of quantum mechanics, the attitude of physicists concerning this situation is now very different from their initial attitude. The duality evident in the wave-particle aspects of electromagnetic radiation is no longer considered unusual because it is now known to be a general characteristic of all physical entities. Furthermore, this duality is no longer considered to represent a problem, since it is possible to reconcile the existence of both aspects with the aid of the theory of quantum mechanics. This theory will be discussed in detail later.

**BIBLIOGRAPHY**


**EXERCISES**

1. Electrons are emitted at thermal velocities from a heated cathode, are accelerated through a voltage drop $V$, and then move in a circle of radius $R$ through a magnetic field of strength $H$ perpendicular to their direction of motion. Derive an expression relating $V$, $H$, and $R$ to $e/m$. Design equipment, based upon this expression, with which it would be possible to make a 1 percent measurement of $e/m$.

2. Prove the statement, made in section 6, that the average kinetic energy of a vibrating system is proportional to the square of the amplitude of its vibration. Do this for the case of a pendulum.

3. Derive an expression for the kinetic energy of the recoiling electron in the Compton scattering process.

4. Derive the Compton equation for the case $\theta = 180^\circ$ by transforming to the CM frame, treating the collision, and then transforming back to the LAB frame.

5. List the experimental evidence, mentioned in section 9, for the properties of electromagnetic radiation that can be explained only by wave motion.

**CHAPTER 4**

The Discovery of the Atomic Nucleus

1. Thomson's Model of the Atom

That electrons are emitted from the metallic cathode of a cathode ray or photoelectric tube implies that the atoms of which the cathode is comprised contain electrons. If so, it is reasonable to assume that all atoms contain electrons. This assumption is attractive because it leads to the simple picture of a positively ionized atom as an atom from which one or more electrons have been removed. It agrees with the experimental observation that the charge of a singly ionized atom is equal in magnitude to the charge of a single electron, or that the charge of a doubly ionized atom is equal to the magnitude of the charge of two electrons, etc.

Additional evidence for the existence of electrons in atoms was soon obtained from the experiments of Barkla and others (1909) concerning the scattering of X-rays by atoms. These experiments will be discussed in Chapter 14, but it is appropriate to mention at this point that the experiments provided an estimate of $Z$, the number of electrons in an atom. It was found that $Z$ is roughly equal to $A/2$, where $A$ is the chemical atomic weight of the atom in question. Another set of experiments which provided a measure of the number of electrons in an atom will be described in this chapter.

Since atoms are normally neutral, they must also contain positive charge equal in magnitude to the negative charge carried by their normal complement of electrons. Thus a neutral atom has a negative charge of magnitude...