is moved, the interference pattern changes: for a stage motion $\Delta z = \lambda/4$
 bright fringes become dark and vice versa, and the original pattern reappears for $\Delta z = \lambda/2$. When the motion is continuous the fringe pattern
 appears to “walk” across the screen and one can count how many fringes have passed by, for a given amount of motion. It is convenient to measure
 $\Delta z$ for $\sim 25$ fringes at a time; this corresponds to motion of $\sim 8$ $\mu$m, which can be adequately resolved by the counter on the translation stage. The
 wavelength is immediately obtained from

$$\lambda = 2(\Delta z/N), \quad (4.44)$$

where $\Delta z$ is the motion of the stage and $N$ the number of fringes that passed by.

### 4.6. THE FABRY–PEROT INTERFEROMETER

In the Michelson interferometer, two coherent waves were made to interfere. In the arrangement introduced by Fabry and Perot a very large (in
 theory infinite) number of waves are made to interfere. Because of the participation of many waves, very sharp contrast between bright and dark
 fringes can be obtained and this results in excellent wavelength resolution.

The Fabry–Perot consists of two mirrors, often parallel plates coated on
 their inner surface to have good reflectivity at the wavelength of interest.
 The spacing, $t$, between the plates is maintained by precision spacers,
 forming an assembly referred to sometimes as an étalon. This is shown
 schematically in Fig. 4.18, where for simplicity we have shown the plates
 as infinitely thin. A ray incoming at an angle $\theta$ with respect to the normal
 after traversing plate 1 will undergo repeated reflections. We label the rays
 emerging from plate 2 by $AB, CD, EF$, etc. The path difference between
 two adjacent rays, say $AB$ and $CD$, is

$$\Delta \ell = BC + CK$$

with $BK$ normal to $CD$. The finite thickness of the plate does not modify
 the above relation. It follows that

$$\Delta \ell = 2t \cos \theta. \quad (4.45)$$

Note that $CK = BC \cos 2\theta$ and $BC \cos \theta = t$; thus, $\Delta \ell = BC(1 + \cos 2\theta) = 2BC \cos^2 \theta = 2t \cos \theta$. Therefore, constructive interference
4.6 The Fabry–Perot Interferometer

Coated surfaces

FIGURE 4.18 The Fabry–Perot interferometer. A ray incident at an angle $\theta$ is shown. For simplicity the mirrors are indicated as infinitely thin. Note that an infinite number of reflections contribute to the transmitted intensity at angle $\theta$.

will occur when the path difference is a multiple of a wavelength

$$2t \cos \theta_n = n\lambda.$$  

(4.46)

Since $\theta_n$ is a small angle, $n$ is a large number of order $n \sim 2t/\lambda$.

The above constructive interference condition holds provided the distance form the étalon to the point of observation is the same for all rays, namely when the observation point is at infinity. To achieve this we use a lens to focus the rays emerging from the étalon onto a screen. For a slightly diverging incident beam one observes a set of rings of radius

$$r_n = f \tan \theta_n \sim f \theta_n,$$  

(4.47)

where $\theta_n$ is determined by Eq. (4.46) and $f$ is the focal length of the lens. Note that the incident beam should not be perfectly collimated but should contain enough angular divergence to support the angles $\theta_n$.

To obtain the spacing between consecutive maxima (fringes) we first note that for $\theta = 0$, the path difference between adjacent beams, measured in wavelengths, is

$$n_0 = 2t/\lambda,$$  

(4.48)
which in general is not an integer. The first observable ring is formed at an angle $\theta_1$ where $n_1$ is the integer closest to (smaller than) $n_0$. Thus

$$n_1 = n_0 - \epsilon \quad 0 \leq \epsilon \leq 1$$

and

$$\epsilon = \frac{2t}{\lambda} (1 - \cos \theta_1) = \frac{4t}{\lambda} \sin^2 \left( \frac{\theta_1}{2} \right). \quad (4.49)$$

As we move out from the center, the $p$th ring corresponds to

$$n_p = (n_0 - \epsilon) - (p - 1). \quad (4.50)$$

Using Eq. (4.50) in Eq. (4.46), recalling the definition of Eq. (4.48), dropping $\epsilon$ with respect to $(p - 1)$ and replacing $2 \sin^2(\theta/2)$ by $\theta^2/2$ we find that the angle of the $p$th ring is

$$\theta_p \simeq \sqrt{\frac{p - 1}{t}} \frac{\lambda}{\theta}, \quad (4.51)$$

applicable for moderately large values of $p$, $p \gtrsim 5$. As an example, if $t = 1$ cm and $\lambda = 633$ nm we have $\lambda/t \simeq 6.3 \times 10^{-5}$ and the $p = 11$ ring will appear at $\theta = 25 \times 10^{-3}$ rads; for a lens with focal length $f = 40$ cm the radius is 1 cm.

Next we calculate the intensity of the rings (fringes) and the contrast between bright and dark fringes. We designate by $T$ the power transmission coefficient of the inner surfaces of the étalon. For simplicity we also assume that both surfaces have the same transmission and reflection coefficients. The power reflection coefficient is $R$, so that in the absence of absorption

$$R + T = 1.$$ 

The amplitude transmission and reflection coefficients are designated by

$$t = \sqrt{T} \quad \text{and} \quad r = \sqrt{R}.$$ 

We also designate the incident intensity by $I_0$ and the amplitude by $A_0$, where $I_0 = \frac{1}{2} A_0^2$. The transmitted ray $B$ will have amplitude

$$A_B = A_0 t^2 e^{i\phi}, \quad (4.52)$$
where $\phi$ is a phase acquired in traversing both plates and the space between them. Ray $D$ will have amplitude

$$A_D = A_B r^2 e^{i2\delta}, \quad (4.53)$$

ray $F$

$$A_F = A_D r^2 e^{i2\delta}, \quad (4.53')$$

and so on. Here the phase angle $2\delta$ is due to the path difference of adjacent rays as they travel between the plates. It follows from Eq. (4.45) that

$$2\delta = 2\pi \frac{2t \cos \theta}{\lambda}. \quad (4.54)$$

From Eqs. (4.53) we see that the amplitude of successive rays decreases by $r^2 = R$, but there is an infinite number of such rays. The amplitude of the transmitted light is

$$A_T = A_0 t^2 e^{i\phi} \sum_{q=1}^{\infty} \left[1 + r^{2q} e^{i2\delta}\right]. \quad (4.55)$$

This geometric series can be easily summed

$$A_T = A_0 t^2 e^{i\phi} \frac{1}{1 - r^2 e^{i2\delta}},$$

and the transmitted intensity

$$I_T = \frac{1}{2} |A_T|^2 = I_0 \frac{T^2}{(1 - R)^2 + 4R \sin^2 \delta}. \quad (4.56)$$

Maxima occur when $\delta$ is an integral multiple of $\pi$, whereas minima occur when $\delta$ is a half-integral multiple of $\pi$. At the maxima

$$I_T = \frac{I_0 T^2}{(1 - R)^2}. \quad (4.57)$$

We see that in the absence of absorption $I_T$ (max) = $I_0$. At the minima

$$I_T = \frac{I_0 T^2}{(1 + R)^2} = I_0 \frac{(1 - R)^2}{(1 + R)^2}, \quad (4.58)$$

showing that very good contrast can be achieved if $R$ is close to 1. Equation (4.56) is plotted in Fig. 4.19 for different values of $R$. 

The bright fringe will reach half its peak intensity when

$$4R \sin^2(\delta/2) = (1 - R)^2$$

or when

$$\delta/2 = \frac{(1 - R)}{2\sqrt{R}}, \quad (4.59)$$

where the small angle approximation was used. The full-width at half-maximum (FWHM) of the fringe is $2\delta/2$. The spacing between adjacent fringes corresponds to a phase angle difference of $2\pi$, and we define the *finesse* of the Fabry–Perot interferometer as the ratio between fringe spacing and the HWHM of the fringe

$$F = \frac{2\pi}{\delta/2} = \frac{\pi \sqrt{R}}{1 - R}. \quad (4.60)$$

For a typical reflectivity $R = 0.98$, the finesse is $F = 155$.

The spacing between bright fringes defines the free *spectral range* of the interferometer. Let the wavelength $\lambda_1$ form its $p$th ring at angle $\theta$, and wavelength $\lambda_2$ form its $(p - 1)$ ring at the same angle. Since these two rings overlap,

$$(n - 1)\lambda_2 = n\lambda_1 \quad \text{or} \quad \lambda_2 - \lambda_1 = \lambda_2/n.$$
However, $n\lambda_1 \sim n\lambda_2 \sim 2t$, so we obtain that

$$\lambda_2 - \lambda_1 = \frac{\lambda_2^2}{2t}.$$ (4.61)

If we express Eq. (4.61) in terms of frequency, $v = c/\lambda$ we find that

$$v_1 - v_2 = \frac{c}{2t}.$$ Namely overlapping rings correspond to the free spectral range already introduced in Eq. (4.12). For instance for $\lambda = 633$ nm and $t = 1$ cm the wavelength spacing is $\delta \lambda = \frac{\lambda^2}{2t} = 0.02$ nm. However, lines between fringes can be resolved if they do not exactly overlap and this depends on the line width, i.e., the finesse of the instrument. Thus, the wavelength resolution is given by

$$d\lambda = \frac{1}{F} (\lambda_2 - \lambda_1) = \frac{1}{F} \frac{\lambda^2}{2t}.$$ (4.62)

For the above example and for $F = 155$, $d\lambda/\lambda \sim 2 \times 10^{-7}$, showing that extremely high resolution can be achieved with a relatively simple apparatus.

Fabry–Perot etalons used in conjunction with lasers are frequently made with two focusing mirrors rather than flat plates. This facilitates the alignment but fixes the free spectral range. They serve as high-resolution filters to select specific wavelengths and as optical “spectrum analyzers,” which are in essence high-resolution scanning spectrometers.