The Concave Diffraction Grating

In 1882 Prof. H. A. Rowland conceived the idea of combining the principle of the plane diffraction grating with the focusing properties of a concave mirror. He found that such concave gratings had the following excellent but simple focusing properties. If a concave grating is placed tangentially to a circle of a diameter equal to the radius of curvature of the grating such that the grating center lies on the circumference, the spectrum of an illuminated point lying on the circle will be focused on this circle (see Fig. 2.1). This circle is known as the Rowland circle and forms the basis of nearly all vacuum spectrographs today.

Rowland further contributed to the use of the concave grating byperfecting the ruling engine. Before this, the ruling of plane gratings was

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Fig. 2.1 The Rowland circle. Radiation from the point E is dispersed and focused by the grating at $\lambda_1$, $\lambda_2$, etc. $\alpha$ and $\beta$ are the angles of incidence and diffraction, respectively.
difficult and often unsuccessful. In his own words Rowland writes [1]:

"One of the problems to be solved in making a machine is to make a perfect screw; and this mechanics of all countries have sought to do for over a hundred years, and have failed. On thinking over the matter, I devised a plan whose details I shall soon publish, by which I hoped to make a practically perfect screw; and so important did the problem seem, that I immediately set Mr. Schneider, the instrument-maker of the University, at work at once."

The "plan" succeeded, and the near perfect screw was made. As a result, Rowland was able to rule excellent gratings with as many as 43,000 lines per inch. His ruling engine is still the prototype of all such machines in operation today. A complete description of Rowland's ruling engine was given by J. S. Ames [2] in 1902. A more recent discussion on ruling engines and the production of diffraction gratings has been given by Harrison [3], who suggested that ruling engines be interferometrically controlled in order to maintain uniformly spaced grooves. For the resolving power of a grating to approach its theoretical limit, it is necessary for a given groove to be ruled to within a $\frac{1}{10}$ of a visible fringe from its proper position relative to the first groove ruled, even although this distance is 20 cm or more. In 1955, Harrison and Stroke [4] reported the first successful use of interferometric control of a ruling engine. An excellent review of high resolution spectroscopy and ruling under interferometric control has been given by Stroke [5].

2.1 THEORY

The theory of the concave grating was developed to a large degree by Rowland himself [6]. He showed that the rulings should be so spaced on the spherical surface as to be equidistant on the chord of the circular arc. Since 1883 there have been numerous contributions to the theory of the concave grating. The most complete developments have been given by Beutler [7] and Namioka [8], who used geometrical optics, and by Mack, Stehn, and Edlen [9] who used physical optics. A rather complete bibliography is given in the papers of Namioka.

With the principles of geometrical optics it is possible to determine the focusing properties of the concave grating as well as the degree of astigmatism present. Setting up a Cartesian coordinate system with the origin $O$ located at the center of the grating rulings, let the $x$ axis be the grating normal with the $z$ axis parallel to the rulings. Let $A(x, y, z)$, $B(x', y', z')$, and $P(u, w, l)$ be points on the entrance slit, image, and grating, respectively (see Fig. 2.2). Then the condition for two rays reflected from adjacent grooves to reinforce at $B$ is that their path difference must equal an integral number of wavelengths. That is, the path difference $= m\lambda$, where $m = 1, 2, 3, \ldots$, etc., is called the spectral order. The path difference for two rays reflected from grooves a distance $w$ apart with a constant groove separation $d$ is equal to $m\lambda w/d$. Thus for reinforcement of all rays at $B$ any arbitrary light path $APB$, where $P$ varies over the grating surface, may be represented by the path function $F$ given by

$$F = AP + BP + \frac{wm\lambda}{d},$$

(2.1)

where

$$(AP)^2 = (x - u)^2 + (y - w)^2 + (z - l)^2,$$

(2.2)

and

$$(BP)^2 = (x' - u)^2 + (y' - w)^2 + (z' - l)^2.$$  

(2.3)

Introducing cylindrical coordinates, we see from Fig. 2.2 that $x = r \cos \alpha$, $y = r \sin \alpha$, $x' = r \cos \beta$, and $y' = r \sin \beta$, where $\alpha$ and $\beta$ are the angles of incidence and diffraction, respectively. The signs of $\alpha$ and $\beta$ are opposite if $A$ and $B$ lie on different sides of the $xz$ plane. Furthermore, since all points such as $P$ lie on a sphere of radius $R$, we have

$$u = R \pm [R^2 - (w^2 + R^2)]^{1/2}.$$  

(2.4)
Only the minus sign is significant in this application. Now converting to cylindrical coordinates and substituting eq. (2.4) for $u$ in eqs. (2.2) and (2.3), we find

$$(AP)^2 = (r - w \sin \alpha)^2 + (z - l)^2 - l^2 \frac{P \cos \alpha}{R} + w^2 \left( \cos^2 \alpha - \frac{r \cos \alpha}{R} \right)$$

$$+ \left( \frac{w^2 + l^2}{4R^2} \right) \left( 1 - \frac{r \cos \alpha}{R} \right) \left( 1 + \frac{w^2 + l^2}{2R^2} + \cdots \right), \quad (2.5)$$

and

$$(BP)^2 = (r' - w \sin \beta)^2 + (z' - l)^2 - l^2 \frac{P \cos \beta}{R} + w^2 \left( \cos^2 \beta - \frac{r' \cos \beta}{R} \right)$$

$$+ \left( \frac{w^2 + l^2}{4R^2} \right) \left( 1 - \frac{r' \cos \beta}{R} \right) \left( 1 + \frac{w^2 + l^2}{2R^2} + \cdots \right). \quad (2.6)$$

If we take the square root of these two equations and insert them into (2.1), the light path function $F$ can be found. According to Fermat's principle of least time, point $B$ is located such that $F$ will be an extreme for any point $P$. Since the points $A$ and $B$ are fixed while $P$ may be any point on the surface of the grating, the conditions for $F$ to be an extreme are

$$\frac{\partial F}{\partial l} = 0 \quad (2.7)$$

and

$$\frac{\partial F}{\partial w} = 0. \quad (2.8)$$

If (2.7) and (2.8) could be satisfied simultaneously by any pair of $l$ and $w$ for the fixed point $B$, then $B$ would be the point of perfect focus. However, a perfect image cannot be obtained from a concave grating. As in the case of a spherical concave mirror, the concave grating will image a point source first into a vertical line (horizontal focus), then into a horizontal line (vertical focus). It can be shown from (2.7) and (2.8) that for $B$ to be the best horizontal focal point, the following condition must hold:

$$\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} + \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} = 0. \quad (2.9)$$

Two solutions of this equation are

$$r = R \cos \alpha, \quad r' = R \cos \beta, \quad \{ \text{or} \} \quad r = \infty, \quad r' = \frac{R \cos^2 \beta}{\cos \alpha + \cos \beta}. \quad (2.10)$$

Equation 2.10 is the equation of a circle expressed in polar coordinates and is known as the Rowland circle. It expresses the fact that diffracted light of all wavelengths will be focused horizontally on the circumference of a circle of diameter $R$ equal to the radius of curvature of the grating, provided that the entrance slit and grating are located on the circle and the grating normal lies along a diameter (see Fig. 2.1). This is the normal condition for observing a spectrum. With vertical entrance slits and vertical rulings it is important that the spectrum be focused in the horizontal plane.

The vertical or secondary foci are more important in the reduction or elimination of astigmatism (see Sect. 2.5). The locus of the vertical foci is given by the equation

$$\frac{1}{r} - \frac{\cos \alpha}{R} + \frac{1}{r'} - \frac{\cos \beta}{R} = 0. \quad (2.12)$$

The solutions of this equation are

$$r = \frac{R}{\cos \alpha}, \quad r' = \frac{R}{\cos \beta}, \quad (2.13)$$

and

$$r = \infty, \quad r' = \frac{R}{\cos \alpha + \cos \beta}. \quad (2.14)$$

Equation 2.13 is the equation for a straight line tangent to the Rowland circle at the normal to the grating. Thus any point on the tangent will be focused vertically and brought to a horizontal astigmatic line on the same tangent.

Equations 2.11 and 2.14 represent the case where the incident light is parallel. If the spectrum is viewed near the normal, then $\cos \beta \sim 1$, both these equations become identical, and $r' = R/(1 + \cos \alpha)$. That is, the vertical and horizontal foci coincide, and the image will be stigmatic. This is the condition for the Wadsworth mounting. When the grating is illuminated close to normal, $r' \sim R/2$.

It can also be shown from (2.7) and (2.8) that for the central ray $AOB$,

$$\left( 1 + \frac{z^2}{r^2} \right) (\sin \alpha + \sin \beta_o) = \frac{m \lambda}{d},$$

and

$$\frac{z}{r} = - \frac{z_o}{r'_o}, \quad (2.16)$$

where $(r_o, \beta_o, z'_o)$ are the coordinates of the image point for the central ray. Equations 2.15 and 2.16 represent, respectively, the grating equation and
the geometrical relation between object point and image point. By letting \( z \) tend to zero in (2.15) we obtain the same grating equation as for a plane grating. In practice, however, \( z^2/r^2 \ll 1 \) and can be neglected. Thus the grating equation for a concave grating can be taken as

\[
\pm m\lambda = d(\sin \alpha + \sin \beta), \tag{2.17}
\]

where the negative signs applies when the spectrum lies between the central image (\( \alpha = \beta \)) and the tangent to the grating (sometimes referred to as the "outside order"). When the spectrum lies between the incident beam and the central image, the positive sign must be used, and the spectrum is referred to as the "inside order".

### 2.2 Dispersion

The dispersion of a grating expresses how the various wavelengths are distributed along the Rowland circle. The angular dispersion is defined as \( d\beta/d\lambda \). This quantity can be easily determined by differentiating (2.17).

For a fixed angle of incidence we have

\[
\frac{\Delta \beta}{\Delta l} = \frac{m}{d \cos \beta}. \tag{2.18}
\]

More frequently we are interested in the actual number of angstroms per mm dispersed along the Rowland circle. This quantity, which is actually the reciprocal of the linear dispersion \( dl/d\lambda \), is called the plate factor. If we rewrite the plate factor in terms of \( \Delta \beta \), we have

\[
\frac{\Delta \lambda}{\Delta l} = \frac{\Delta \lambda}{\Delta \beta} \frac{\Delta \beta}{\Delta l}. \tag{2.19}
\]

From Fig. 2.3 we see that \( R\Delta \beta = \Delta l \), where \( R \) is the radius of the concave grating. Therefore, (2.19) becomes

\[
\frac{\Delta \lambda}{\Delta l} = \frac{1}{R(\Delta \beta/\Delta \lambda)}. \tag{2.20}
\]

We can replace \( \Delta \) in the limit by the differential \( d \). Thus from (2.18) and (2.20), we obtain

\[
\text{PLATE FACTOR} \quad \frac{d\lambda}{dl} = \frac{d \cos \beta}{mR}
\]

\[
= \frac{\cos \beta}{mR(1/d)} \times 10^4 \quad \text{Å/mm}, \tag{2.21}
\]

when \( R \) is measured in meters and \( 1/d \) is the number of lines per mm.

### 2.3 Resolving Power

The resolving power and dispersion are closely related quantities. While the dispersion determines the separation of two wavelengths, the resolving power determines whether this separation can be distinguished. Each monochromatic beam itself forms a diffraction pattern, the principal maxima of which are represented by the order number \( m \). Between such maxima, secondary maxima exist whose intensities decrease as the number of ruled lines \( N \) exposed to the incident radiation increase. In practice, these secondary maxima are very much weaker than the principal maxima. The angular half-width of a principal maximum \( \Delta \beta \) is the angular distance between the principal maximum and its first minimum and in a plane grating is given by

\[
\Delta \beta = \frac{\lambda}{N d \cos \beta}. \tag{2.22}
\]

This width then provides a theoretical limit to the resolving power of a grating. If we use Rayleigh's criterion, two lines of equal intensity will
Table 2.1 Resolving Power of the Concave Grating as a Function of Grating Width

<table>
<thead>
<tr>
<th>Width of Grating $W$</th>
<th>Resolving Power $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W = W_{\text{opt}}$</td>
<td>$0.92 W_{\text{opt}} (m/d)$</td>
</tr>
<tr>
<td>$W &gt; W_{\text{opt}}$</td>
<td>$0.75 W_{\text{opt}} (m/d)$</td>
</tr>
</tbody>
</table>

Table 2.2 Optimum Width $W_{\text{opt}}$ (cm) for a 1-Meter Grating
(for gratings with radius $R$ cm multiply $W_{\text{opt}}$ by $[R/100]^3$)

<table>
<thead>
<tr>
<th>$m/d = 6000$ cm$^{-1}$</th>
<th>$m/d = 12000$ cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda (\text{Å})$</td>
<td>$\lambda (\text{Å})$</td>
</tr>
<tr>
<td>10 100 500 1000 2000</td>
<td>10 100 500 1000 2000</td>
</tr>
<tr>
<td>6000 cm$^{-1}$</td>
<td>12000 cm$^{-1}$</td>
</tr>
</tbody>
</table>

Whether this theoretical limit is achieved will depend on the actual quality of the grating. That the resolving power increases with the order number $m$ is clear from the fact that the angular half width of the principal maxima is practically independent of $m$, whereas the angular dispersion essentially increases linearly with $m$. The $\cos \beta$ term in these two relations varies slowly with $m$.

Equation 2.23 is true strictly for the plane diffraction grating. However, it is also true for concave gratings when the width of the grating is less than than a certain optimum value. It has been shown by Namioka [8] and by Mack, Stehn, and Edlén [9] that for a concave grating, the angular half-width of a principal maximum deviates from that given by (2.22), and that the diffracted minima do not reach zero. Thus they introduced a modified Rayleigh’s criterion. The basic change from Rayleigh’s definition is that they do not require the maximum of one line to fall on the first minimum of the other line. However, the ratio of the minimum intensity of the composite structure, as shown in Fig. 2.4b, to that of either maximum remains the same as in the Rayleigh definition, namely, $8/\pi^2$. Expressed formally, the modified criterion states that two lines of equal intensity will be just resolved when the wavelength difference between them is such that the minimum total intensity between the lines is $8/\pi^2 (= 0.8106)$ times as great as the total intensity of both at the central maximum of either of the lines. Using this new criterion, they have shown that the resolving power of a concave grating is equal to $mN$ when the width $W$ of the grating illuminated is less than or equal to $W_{\text{opt}}/1.18$. The optimum width $W_{\text{opt}}$ is discussed below. As the width of the grating increases beyond $W_{\text{opt}}/1.18$, the resolving power still increases (but not so rapidly as $mN$) until it reaches a maximum at a width $W = W_{\text{opt}}$. This optimum resolving power, $R_{\text{opt}}$, is then equal to $0.92 W_{\text{opt}} (m/d)$. For $W > W_{\text{opt}}$, the resolving power tends to oscillate about a mean value such that for $W > W_{\text{opt}}$, $R = 0.75 W_{\text{opt}} (m/d)$. The above results are summarized in Table 2.1, and typical values of $W_{\text{opt}}$ are listed in Table 2.2. The above discussion applies to the case when the center of the illuminated slit lies on the Rowland circle, known as in-plane mounting. For off-plane mountings, such as the Eagle mounting, $R_{\text{opt}} = 0.95 W_{\text{opt}} (m/d)$ (see Chapter 3).
From physical optics, it can be shown that the optimum width of a concave grating is given by

\[ W_{\text{opt}} = 2.51 \left[ \frac{R^2 \lambda \cos \alpha \cos \beta}{\sin^2 \alpha \cos \beta + \sin^2 \beta \cos \alpha} \right]^{1/4}. \]  

(2.24)

This is essentially the same result as derived from geometrical optics with the exception that the numerical constant is then equal to 2.38. Values of \( W_{\text{opt}} \) are given in Table 2.2 as a function of wavelength and angle of incidence for a 1 m grating with \( m/d = 6000 \text{ cm}^{-1} \) and 12000 cm\(^{-1}\). To obtain the value of \( W_{\text{opt}} \) for any radius \( R \) cm, multiply the values in Table 2.2 by \((R/100)^{1/4}\).

Because \( W_{\text{opt}} \) decreases when either \( \alpha \) or \( \beta \), or both, increase, then the resolving power will also decrease. Thus it is always desirable to work with the inside spectrum in order to minimize \( \beta \). For example although the dispersion tends to infinity as \( \beta \rightarrow \pi/2, R \rightarrow 0 \). For high resolution it is thus not desirable to use greater angles of incidence and diffraction than are necessary for sufficient reflection.

### 2.4 ASTIGMATISM

The major aberration of a concave mirror is astigmatism, and this imperfection is inherited by the concave diffraction grating. The theory of the astigmatism of a concave grating was first developed by Runge and Mannkopf [10]. More recently it has been dealt with in detail by Beutler [7] and Namioka [8].

Astigmatism results in a point on the slit being imaged into a vertical line; that is, focusing is achieved only in the horizontal plane. The length of the astigmatic image \( z \) is given by [7,8]

\[ z = \left[ \frac{l \cos \beta}{\cos \alpha} \right] + L[\sin^2 \beta + \sin \alpha \tan \alpha \cos \beta], \]  

(2.25)

where the first term gives the contribution due to an object slit (or entrance slit) of finite vertical length \( l \), and the second term is the astigmatism produced by a point on the object slit. \( L \) represents the length of the ruled lines illuminated. As can be seen from eq. (2.25), the image becomes less astigmatic for near normal incidence and quite stigmatic for \( \alpha = \beta = 0^\circ \). Astigmatism is most severe at grazing angles. Figure 2.5 gives the lengths of the astigmatic images of a point source on the slit in units of length of the rulings for a 600 line per mm grating. Thus for an angle of incidence \( \alpha = 88^\circ \) with rulings 2 cm long, the astigmatic image of a point source at 500 Å is 16 cm long.

In general, the astigmatic image is not a straight line. The curvature of the images has been studied by Beutler [7], who identified two types of curvature. The curved spectral lines caused by the astigmatism of a point source at the entrance slit was called the astigmatic curvature. Another kind of curvature is caused by the finite length of the entrance slit when illuminated, and was called by Beutler the enveloping curvature. Referring to Fig. 2.6, let the point \( A \) on the Rowland circle be focused as the astigmatic image \( BCD \). The length of the chord \( x \) is given by (2.25). The sagittal distance \( x \) as calculated by Beutler for the case of astigmatic curvature is given by

\[ x = \frac{z^2}{8R} \Psi^2, \]  

(2.26)

where \( R \) is the diameter of the Rowland circle and \( \Psi^2 \) is the angular function

\[ \Psi^2 = \frac{1}{1^2} \left[ \sin \alpha \tan^2 \alpha - \sin \beta + \frac{\tan \beta}{\cos \beta} (1 - 1')^2 \right]. \]  

(2.27)
THE CONCAVE DIFFRACTION GRATING

where $\Gamma = (\sin^2 \beta + \sin \alpha \tan \alpha \cos \beta)$. Equation 2.26 represents a parabola. However, it also approximates the equation for a circle of radius $r$ when $r^2 \gg x^2$, namely, $x^2 = 8\alpha r$. Thus the curve $BCD$ can be represented by an arc of a circle of radius

$$r = \frac{R}{\Psi}. \quad (2.28)$$

The angular function $\Psi$ is plotted in Fig. 2.7 as a function of $\alpha$ and $\beta$. When $\Psi$ is positive, the curvature is concave towards the entrance slit as shown in Fig. 2.6. For the central image, $\Psi = 1/\sin \alpha$. Thus at grazing incidence, the radius of curvature of the central image is approximately equal to the radius of the grating.

The expression for the enveloping curvature as given by Beutler appears to be in error according to Welford [11]. The radius of curvature given by Welford is

$$r_x = \frac{R}{\Phi}. \quad (2.29)$$

where

$$\Phi = \frac{(\sin \alpha + \sin \beta)}{\cos^2 \beta}. \quad (2.30)$$

For the central image, the curvature $(1/r_x)$ is zero. The function $\Phi$ is plotted in Fig. 2.8. From Figs. 2.7 and 2.8, it can be seen that for the inside spectra, the astigmatic curvature is generally greater than the enveloping curvature in the vacuum uv. Although the astigmatic curvature is greatest near normal incidence, its effect is not important since the height of the astigmatic image is nearly zero. The enveloping curvature is very small near normal incidence thus the spectral lines are effectively straight. As the angle of incidence increases, the length of the astigmatic image increases more rapidly than the decrease in the astigmatic curvature; that is, the sagittal distance $x$ increases with the angle of incidence. The curvature of the image lines limits the wavelength resolution obtainable by a monochromator that uses straight parallel exit slits. For example, for a straight exit slit of 1 cm in length in a Seya-monochromator ($\alpha = 30^\circ15'$), the value of $x$ for the central image is 22 $\mu$, considering the astigmatic curvature only. When the exit slit width is very much greater than $x$, the
resolution impairment due to the curvature of the spectral lines is negligible and only becomes important when $x$ and the slit width are of the same order of magnitude.

Astigmatism can be tolerated in spectroscopy since only horizontal focusing is required to separate the various wavelengths. However, it reduces the light intensity per unit area of the image and imposes strict focusing conditions to produce maximum resolution. Techniques for the reduction or elimination of astigmatism are discussed below.

2.5 STIGMATIC IMAGES

Under certain conditions it is possible to produce vertical focusing with a concave grating; that is, horizontal lines will be imaged as horizontal lines. This geometrical relation between object and image was first pointed out by Sirks [12]. He noted that a point source of light placed at $N$, where the normal to the grating meets the Rowland circle (Fig. 2.9), would be brought to a horizontal focus at $S$ and to a vertical focus along $ABC$, an extension of the tangent to the Rowland circle at $N$. Thus a horizontal
line of narrow rectangular aperture placed along $ABC$ will come to a vertical focus at $N$. The stigmatic conditions hold strictly for rays diffracted along the normal and approximately for a short distance on either side of the normal. These conditions led Sirks to propose the following geometrical arrangement for the superposition of a comparison spectrum, a technique previously suitable only with stigmatic spectographs. Placing a short but rather broad prism (2 or 3 mm in height) along $ABC$, he reflected sunlight through the slit $S$ into the spectrograph, which gave a narrow solar spectrum with perfectly defined edges passing through the center of the field. At the same time, light from a sodium flame placed at $T$ passed over the top and bottom of the prism to form an astigmatic spectrum above and below the solar spectrum. This arrangement covers a very small spectral region at one setting, and, since the spectrum must be viewed along the normal to the grating, $S$ must be moved around the Rowland circle to observe another spectral region. The secondary focal plane $ABC$ is always positioned behind $S$ at a distance

$$SB = R \sin \alpha \tan \alpha.$$  

Sirks' focusing condition is a special case of the vertical focusing conditions discussed in Sect. 2.1. It was noted there that any point on the tangent to the Rowland circle passing through the normal to the grating was focused in the vertical plane at some other point on the tangent. $N$ is the only point common to both the focusing conditions. Thus a line illuminated at $ABC$ will be focused vertically at $N$, while the vertical slit at $S$ will make the line source appear as if it were a point on $S$. It will thus be focused horizontally as well.

We return to the Rowland circle mount. It is important to know where point light sources or their virtual images must be located in order to produce stigmatic images on the Rowland circle. We will call this position the vertical focal point. Equation 2.12 represents the general equation for vertical focusing. For the image to be located on the Rowland circle, we must put $r' = R \cos \beta$ in (2.12). Thus the locus $r$ for such light sources is given by

$$\frac{1}{r} - \frac{\cos \alpha}{R} + \frac{1}{R \cos \beta} = 0,$$

where $r$ is measured from the grating center. Therefore,

$$r = R \left( \frac{1}{\cos \alpha - \sin \beta \tan \beta} \right).$$  

The distance $l$ between the primary slit $S$ and the vertical focal point is given by $l = r - r_o$, where $r_o$ is the distance between $S$ and the grating and is equal to $R \cos \alpha$; that is,

$$l = R \left( \frac{1}{\cos \alpha - \sin \beta \tan \beta} - \cos \alpha \right).$$  

For $\beta = 0$, (2.33) reduces to $l = R \sin \alpha \tan \alpha$, which is identical to (2.31).

The fact that a stigmatic image for a given wavelength can be achieved using a concave diffraction grating has increased the usefulness of the concave grating. An important application of (2.33) has been in solar spectroscopy using rocket-borne spectographs [13–16]. Since the light source (the sun in this case) is at infinity, then in (2.33), $l = \infty$. This will be true when

$$\cos \alpha = \sin \beta \tan \beta.$$

From the grating equation, we have the further condition that

$$\frac{m \lambda}{d} = \sin \alpha + \sin \beta.$$

For a given order number, wavelength, and grating spacing these two equations can be solved simultaneously for $\alpha$ and $\beta$. Consider the Lyman-$\alpha$ line at 1215.7 Å imaged in the first order by a 600 line per mm grating. Then to produce a stigmatic image of Lyman-$\alpha$ on the Rowland circle with parallel incident light, we find

$$\alpha = 49.5^\circ \quad \text{and} \quad \beta = -43.4^\circ.$$

In the more general case a concave mirror may be used to focus the object onto the entrance slit of the spectrograph [14,16]. This has the added advantage of increasing the light-gathering power of the instrument. Figure 2.10 shows a typical arrangement for the removal of astigmatism at some wavelength for a grazing incidence mount. The mirror must provide a horizontally focused image at $S_w$, the entrance slit of the spectrograph, and a vertically focused image at $L$, the vertical focal point for the grating. The distance $GL$ is given by (2.32). As can be seen from (2.32), when $\alpha$ and $\beta$ are large, $r$ is negative, and the point $L$ is a virtual focal point. However, the above arrangement to eliminate astigmatism applies whether $r$ is negative or positive. A toroidal mirror can be used instead of a spherical mirror giving more freedom in the choice of the magnitude of the optical parameters. The radii of the toroidal mirror and its position relative to the spectrograph are given by the following relations:

**HORIZONTAL FOCUS**

$$\frac{1}{s} + \frac{1}{s_h} = \frac{2}{r_h \cos \varphi},$$  

**VERTICAL FOCUS**

$$\frac{1}{s} + \frac{1}{s_v} = \frac{2 \cos \varphi}{r_v},$$
where \( r_s \) and \( r_t \) are the radii in the plane and at right angles to the plane of the Rowland circle, respectively. The distance \( s \) denotes the object distance of the aperture or slit \( S_t \) and \( \phi \) is the angle of incidence. The combination of a toroidal mirror and a spherical concave grating provides the best method for reducing astigmatism over a wide wavelength range for a grazing incidence spectrograph. For other angles of incidence, typically the Seya-Namioka mount, an ellipsoidal or toroidal grating is better. This is discussed further in the next section.

When working in the higher orders of a grating spectrograph, it is desirable to eliminate overlapping orders. In the vacuum ultraviolet region this can be achieved by using a foregrating [17,17a]. When a spherical or toroidal foregrating is used in place of the mirror in the above discussion, astigmatism is removed at one wavelength and reduced in the vicinity of that wavelength. It should be noted that the foregrating system consisting of slits \( S_t \) and \( S_s \) and the foregrating all lie on a Rowland circle of diameter equal to the radius of the foregrating (see Fig. 2.10). \( S_s \) is common to both grating systems. Equation 2.33 is applied first to the main spectrograph to locate its vertical focal point. Then it is applied to the foregrating system so that \( S_t \) is imaged vertically at the vertical focal point of the main spectrograph. That is, the position \( L \) in Fig. 2.10 is the vertical focal point for both the foregrating and the main grating system. The remaining parameters of the foregrating are determined from the grating equation \( m\lambda = d(\sin \alpha + \sin \beta) \).

### 2.6 Ellipsoidal and Toroidal Gratings

Although no ellipsoidal gratings exist at present, their properties are promising and will be discussed briefly.

The theory of the ellipsoidal diffraction grating has been developed by Namioka [18]. The grating equations are identical to (2.15) and (2.16) for the spherical grating. The focal condition is given by

\[
\cos^2 \alpha - \frac{a}{b^2} \cos \alpha + \frac{\cos^2 \beta}{r'} - \frac{a}{b^2} \cos \beta = 0, \tag{2.36}
\]

where \( a, b, \) and \( c \) are the lengths of the semi-axes of the ellipsoid in the directions of the \( x, y, \) and \( z \) axes, respectively. The other symbols and the geometrical arrangement are as shown in Fig. 2.2 for the spherical grating. A solution of (2.36) is

\[
r = \frac{b^2}{a} \cos \alpha \quad \text{and} \quad r' = \frac{b^2}{a} \cos \beta. \tag{2.37}
\]

Equation 2.37 represents a circle of diameter \( b^2/a \) in the \( xy \) plane similar to the Rowland circle.

The stigmatic condition for an ellipsoidal grating is given by

\[
\frac{c^2}{b^2} = \cos \alpha \cos \beta, \tag{2.38}
\]

which is shown graphically in Fig. 2.11. When (2.38) does not hold, the length of the astigmatic image of a point source is given by

\[
z = L(1 + \sec \alpha \cos \beta) \left| 1 - \frac{b^2}{c^2} \cos \alpha \cos \beta \right|. \tag{2.39}
\]

The optimum grating width is

\[
W_{\text{opt}} = 2.12 \left[ \frac{2Rd}{m} \tan \left( \frac{\alpha + \beta}{2} \right) \frac{\cos \alpha \cos \beta}{\left| 1 - (R/a) \cos \alpha \cos \beta \right|} \right]^{1/4}, \tag{2.40}
\]

where \( R = b^2/a \) is the diameter of the equivalent Rowland circle. In the region where

\[
\cos \beta \leq \frac{2c^2}{b^2 + c^2} \sec \alpha, \tag{2.41}
\]
is valid, and when $a = b \neq c$ or $a = c \neq b$, the ellipsoidal grating gives less astigmatism than the spherical grating. However, for $b = c$, nothing is gained.

In theory the ellipsoidal grating has a better optical performance than the spherical grating both at grazing incidence and in the Seya–Namioka mounting. However, it is less efficient in reducing astigmatism at grazing incidence than the combination of toroidal mirror and spherical grating.

The reduction of astigmatism has been considered theoretically by several authors. Haber [19] has presented the theory of the toroidal grating, showing that it is free from astigmatism at two wavelengths with practically no astigmatism in the vicinity of those wavelengths. Greiner and Schäffer [20] have discussed astigmatism caused by both spherical and toroidal gratings, while Sakayanagi [21] has considered the reduction in astigmatism using a spherical concave grating ruled with circular grooves.

Fig. 2.11 The function $c/b = (\cos \alpha \cos \beta)^{1/2}$ plotted as a function of $\alpha$ and $\beta$. This is the stigmatic condition for an ellipsoidal grating. $b$ and $c$ are the lengths of the semi-axes of the ellipsoid in the direction of the $z$ and $y$ axis, respectively, both measured in the $xy$ plane (courtesy T. Namioka [18]).

Schönheit [21a] has successfully produced and used a toroidal grating for the reduction of astigmatism in a Seya–Namioka monochromator. The radii of the grating can be determined from the equations for horizontal and vertical focusing, namely,

$$\frac{\cos^2 \alpha}{r} + \frac{\cos^2 \beta}{r'} - \frac{\cos \alpha}{R_h} - \frac{\cos \beta}{R_h} = 0$$

and

$$\frac{1}{r} + \frac{1}{r'} - \frac{\cos \alpha}{R_o} - \frac{\cos \beta}{R_o} = 0,$$

where $R_h$ and $R_o$ are the radii of the grating in the plane and at right angles to the plane of the Rowland circle, respectively. For the central image, $\alpha$ is equal to $\beta$ and $r = r'$. Substituting these values into (2.9) and (2.12), we obtain

$$R_o = \frac{R_h}{\cos^2 \alpha_o},$$

where $\alpha_o$ is the angle of incidence when viewing the central image. In the Seya mount, $\alpha_o \sim 35^\circ$, hence $R_o/R_h \sim 0.671$, which is the ratio used by Schönheit. For small rotations of the grating, the images remain approximately stigmatic.

### 2.7 Grating Efficiency

The efficiency of a grating at a given wavelength can be defined as the percentage of the incident radiant flux returned by the grating into a given spectral order.

The groove separation of a diffraction grating influences the angular dispersion of the diffracted radiation. The groove shape, on the other hand, controls the amount of radiation concentrated into a given order. The concentration of radiation into any desired spectrum was first discussed by Lord Rayleigh in 1888. However, it was not until 1910 that the first successful grating was ruled by R. W. Wood [22] with grooves of controlled shape. These gratings were called *echellette* gratings. Because it is now standard practice to rule gratings with grooves of controlled shape, the term *echellette* has been dropped. In its place we sometimes refer to a grating as being blazed at a given wavelength. A discussion of the echelle grating is given in Chapter 3.

Figure 2.12 shows a cross section of a typical blazed grating. Let $N$ be the normal to the macroscopic surface of the grating while $N'$ denotes the normal to the facets. The grooves are cut such that the facets make an angle $\theta$ with the grating surface. The angles $\alpha$ and $\beta$ are, as before, the...
angles of incidence and diffraction. The principle of concentrating radiant energy into a given wavelength is that this wavelength must be diffracted in a direction which coincides with the direction of the specularly reflected beam from the surface of the facet. Referring to Fig. 2.12, this condition is expressed by

\[ \alpha - \theta = \beta + \theta, \]

hence

\[ \theta = \frac{\alpha - \beta}{2}. \]  

(2.42)

However, for Fig. 2.12,

\[ m\lambda = d \sin \alpha - \sin \beta. \]  

(2.43)

Eliminating \( \beta \) from (2.42) and (2.43), we obtain the wavelength \( \lambda_{\text{blaze}} \) for which the grating is blazed, namely,

\[ m\lambda_{\text{blaze}} = 2d \sin \theta \cos (\alpha - \theta). \]  

(2.44)

For normal incidence, \( \alpha = 0 \), and (2.44) becomes

\[ m\lambda_{\text{blaze}} = d \sin 2\theta. \]  

(2.45)

Equation 2.44 is quite general, provided the sign of \( \theta \) is taken to be positive when it is on the same side of the normal as \( \alpha \), and negative if it is on the opposite side. It can be seen from (2.44) or (2.45) that a grating blazed for 2000 Å in the first order \((m = 1)\) is also blazed for 1000 Å in the second order and for 500 Å in the fourth order, etc. Under the conditions of illumination shown in Fig. 2.12, the concentration of radiation can only occur in the inside spectrum. Should the radiation be incident on the opposite side of the normal \( N \) or, equivalently, if the grating is rotated through 180°, \( \theta \) will be negative and the blaze will occur at the same wavelength, but in the outside spectrum. When the grating is illuminated in this direction, however, a fraction of the diffracted beam will always strike the steep edge of the grooves, thus producing some scattered radiation and diminishing the intensity of the diffracted beam. The grating is most efficiently used with the radiation incident "uphill," as shown in Fig. 2.12. Bausch and Lomb place an arrow on the back of their gratings that points towards the blaze angle. Therefore, the arrow should point towards the incoming radiation. Figure 2.13 shows an electron micrograph of test rulings made by the Bausch and Lomb Company on an aluminized mirror. The shadow cast by a metal vapor incident obliquely on an asbestos fiber reveals the shape of the grooves. The white areas of the photograph indicate the shadow of the asbestos rod. Anderson et al.
[23] have described the Bausch and Lomb shadow-casting technique and
the method for analyzing the groove shape from electron micrographs.
Figure 2.14b is a schematic of the photograph shown in Fig. 2.13 and
indicates the dimensions to be used in determining the blaze angle \( \theta \).
Figure 2.14c shows a section through the fiber and grating cut parallel to
the rulings; the fiber rod is assumed to be at right angles to the rulings and
of uniform diameter \( w \). The direction of the metal vapor is shown by the
oblique arrows incident at an angle \( \varphi \) to the normal of the grating surface.
If this particular section is taken through the bottom of the groove, then
the height \( h \) represents the height of the grooves shown in Fig. 2.14c. The
metal vapor strikes the top of the groove at a distance \( x \) from the center
of the fiber and strikes the bottom of the groove at a distance \( y \) from
the center of the fiber. From Fig. 2.14a, the height of the grooves is given by

\[
\begin{align*}
  h &= w \left( \frac{y}{x} - 1 \right) \left( 1 + \frac{1 - \sin \varphi}{2 \sin \varphi} \right).
\end{align*}
\]  

(2.46)

The blaze angle can then be determined from \( \tan \theta = h/d \). The term
\( (1 - \sin \varphi)/(2 \sin \varphi) \) can be neglected when \( \varphi \) is greater than about 65°.
The angle of shadow-casting \( \varphi \) can be determined approximately from the
relation \( \tan \varphi \approx x/w \). To determine the angle \( \varphi \) more precisely, Anderson
et al. placed a latex sphere on the surface of the grating before shadow
casting. Then from the ratio of the length to the width of the shadow of
the sphere, they were able to determine \( \tan \varphi \). Analyzing Fig. 2.13, we
find that the blaze angle is about 4.5°, the step height approximately
1300 A, and the angle of shadow-casting about 77°. The scale of the
photograph can be determined from the known groove separation \( d \), which
in this case is equal to 1.67 \( \mu \) (600 grooves per mm). The electron micro-
graphs show that grooves can be ruled to conform with the ideal shape
shown in Fig. 2.12. It is very revealing to see the surface of the ruled and
unruled areas under the high magnification produced by the electron
microscope. As can be seen from Fig. 2.13, the unruled area of the
evaporated aluminum coating is quite rough, whereas the ruled area,
especially at the bottom of the groove, is considerably smoother. Replica
gratings made from such masters often show superior performance since
the rough edges at the top of the grooves in the master are at the bottom
of the grooves in the replica.

For further discussions on the efficiencies and production of diffraction
gratings, the reader is referred to references 24 to 31.

In some investigations, it is necessary to know the actual efficiency of
a grating as a function of wavelength. This knowledge is necessary, for
example, in determining the spectral energy distribution from sources
such as the sun or high temperature plasmas. The measurement of
grating efficiencies can be readily achieved using the experimental arrange-
ment shown in Fig. 2.15 [32]. Other arrangements have been described
in the literature using polarized and unpolarized incident radiation [33–35].
In this particular arrangement, radiation from the exit slit of a mono-
chromator enters the test chamber and is baffled in order to illuminate a
small portion of the grating surface. By changing the position of the
baffle, the efficiencies of different areas of the ruled surface can be
studied.

To obtain the amplitude of the various diffracted orders, the photom-
multiplier is rotated about the grating. The output current and position
of the photomultiplier relative to the incident beam is recorded when the
output current reaches a maximum value. The grating is then moved out
of the path of the light beam by raising it vertically to allow the irradiance
of the incident beam to be measured. The ratios of the output currents
due to reflected orders to that due to the incident beam gives the required
efficiencies.
Figures 2.16 and 2.17 illustrate typical diffraction patterns found by rotating the photomultiplier about the grating. Figure 2.16 was obtained using a 600 line per mm grating while a 1200 line per mm grating was used in Fig. 2.17. The abscissae are expressed in degrees zeroed on the central image, labelled CI, whereas the ordinate axes represent the percent of the incident radiant flux which is reflected from the grating into the various orders. Figure 2.17 also shows the development of a blaze at 686 Å for the inside first order.

Figure 2.18 shows the efficiencies of the inside first order for three areas of the 1200 line per mm grating as a function of wavelength. Figure 2.19 is a plot of the ratio of the inside first order intensities to that of the sum of all orders, that is, to the total reflectance, for the 1200 line per mm grating. The maxima of such curves represent the wavelength at which the grating is blazed. The curves labelled 1, 2, and 3 refer to the three portions of the grating studied. The dashed curve is the ratio of the inside second order to that of the total reflectance. The maximum of this curve indicates a blaze at 650 Å, approximately half the wavelength at the maximum of the solid curve 3. From Fig. 2.19 it can be seen that the blazed wavelength shifts towards shorter wavelengths as the area of the grating illuminated changes from area 3 toward area 1. Thus, the ruling of this grating apparently produced a continuously changing blaze angle. To maintain a more constant blaze angle over the entire surface of a grating it should be ruled in three portions, the angle of the ruling diamond being reset after each ruling. Such tripartite gratings are available commercially. The resolution of such gratings, however, cannot be greater than that of a single ruled portion.

This discussion is applicable to the study of grating efficiencies at any angle of incidence. However, for radiation shorter than approximately
300 Å the efficiencies of most gratings used at normal incidence are poor and, generally, grazing incidence techniques are employed. Tomboulian and co-workers [36,37] have discussed a method for comparing the reflecting power of concave gratings in the soft x-ray region.

There is some doubt whether the efficiency of a grating used at grazing incidence can be increased by shaping the grooves. In the past, best results appear to have been obtained using gratings lightly ruled on glass such that the area between the grooves is untouched and is extremely smooth [25,26]. The principle of this argument is that the efficiency of a grating will increase when the radiation is incident at a sufficiently large angle of incidence to experience total reflection from the surface. In this case the reflecting surface must be very smooth. The principle of a controlled groove shape is still valid at these extreme angles and would be expected to improve the grating efficiency. However, the problem may be to

rule the grating such that the reflecting facets have the required degree of smoothness. If this can be achieved, a blazed grating might be expected to be superior at grazing incidence angles. Gabriel et al [38] have, in fact, obtained a factor of 2 in efficiency using a platinized replica grating with a blaze angle of 1.5° compared to a lightly ruled glass original grating. They used an angle of incidence of 88° with the radiation incident “uphill” as shown in Fig. 2.12. At this angle of incidence, the grating will be blazed for 53 Å. It is unclear, however, whether the increased efficiency was due to the blazed grating or the high reflectance of the platinum overcoating.

As mentioned above, the efficiency of a grating is also influenced by the reflectance of the material in which the grating is ruled or has been coated. Watanabe [39], using a platinized capillary in a hydrogen dc light source,
2.8 REFLECTIVE COATINGS

From the elementary theory of metals, the critical angle of reflection for radiation of wavelength $\lambda$ is given by

$$\sin \theta_c = \lambda \left( \frac{e^2}{mc^2} \frac{N}{\pi} \right)^{1/2},$$

(2.47)

where $\theta_c$ is the grazing angle of incidence, and $N$ is the number of electrons per unit volume. Thus, for a given wavelength, total reflection occurs at all grazing angles up to a certain maximum $\theta_c$ at which point the reflectance drops rapidly. Conversely, for a given grazing angle of incidence, $\lambda$ in (2.47) represents the minimum wavelength which is reflected, all wavelengths longer than this minimum being totally reflected. Inserting the values of the constants in (2.47), we obtain

$$\lambda_{\text{min}}(\AA) = (3.33 \times 10^{14})N^{-1/2} \sin \theta.$$

(2.48)

For a glass or aluminized grating, $N^{1/2} = 8.8 \times 10^{11}$, therefore for small $\theta$,

$$\lambda_{\text{min}}(\AA) = 6.6 \theta^0.$$

(2.49)

Thus for a grazing angle of incidence of $1^\circ$, the minimum reflected wavelength is 6.6 Å. Of course, the theory does not hold exactly. Nevertheless, it can be used to give an order of magnitude estimate. Table 2.3 lists the value of $\lambda_{\text{min}}$ as a function of the grazing angle $\theta$ for several materials used for overcoating gratings along with two common impurities which are often present on the surface of gratings, namely, oil and oxides [41].

Table 2.3 The Minimum Wavelength Reflected for a Given Grazing Angle of Incidence for Several Grating Materials and Impurities

<table>
<thead>
<tr>
<th>Material</th>
<th>Density</th>
<th>$N$</th>
<th>$N^{1/2}$</th>
<th>$\lambda_{\text{min}}(\AA)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentadecane (oil)</td>
<td>0.77</td>
<td>$27 \times 10^{22}$</td>
<td>$5.2 \times 10^{11}$</td>
<td>641 sin $\theta$</td>
</tr>
<tr>
<td>Glass</td>
<td>2.6</td>
<td>$78 \times 10^{22}$</td>
<td>$8.8 \times 10^{11}$</td>
<td>379 sin $\theta$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>$78 \times 10^{22}$</td>
<td>$8.8 \times 10^{11}$</td>
<td>379 sin $\theta$</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>3.9</td>
<td>$115 \times 10^{22}$</td>
<td>$10.7 \times 10^{11}$</td>
<td>312 sin $\theta$</td>
</tr>
<tr>
<td>Silver</td>
<td>10.5</td>
<td>$276 \times 10^{22}$</td>
<td>$16.6 \times 10^{11}$</td>
<td>201 sin $\theta$</td>
</tr>
<tr>
<td>Gold</td>
<td>19.3</td>
<td>$466 \times 10^{22}$</td>
<td>$21.6 \times 10^{11}$</td>
<td>154 sin $\theta$</td>
</tr>
<tr>
<td>Platinum</td>
<td>21.4</td>
<td>$514 \times 10^{22}$</td>
<td>$22.7 \times 10^{11}$</td>
<td>147 sin $\theta$</td>
</tr>
<tr>
<td>Iridium</td>
<td>22.4</td>
<td>$542 \times 10^{22}$</td>
<td>$23.3 \times 10^{11}$</td>
<td>143 sin $\theta$</td>
</tr>
</tbody>
</table>
coating of oil, it would seriously affect the reflectance of the grating near its short wavelength reflectance limit. This effect has been observed in practice.

The reflectances of several elements have been measured by Hendrick [42] at 8.32 Å as a function of the grazing angle of incidence. His results for Al and Au are shown in Fig. 2.20. The critical angle of reflectance is generally taken to be the angle at which the slope of the reflectance curve is a maximum. For Al, Hendrick measured a critical angle of 18.7 milliradians, and for Au, a value of 37.2 milliradians. Similar measurements have been made by Johnson and Wuerker [43] at 44.6 and 114 Å. Their results for gold are shown in Fig. 2.21 and agree very well with those of Lukirskii, Savinov, and Shepelev [44]. Since gold has many absorption edges in the wavelength range 1 to 175 Å, it is possible that observed spectra may show false structure in the vicinity of these edges. However, a study by Lukirskii, Zimkina, and Brytov [45] showed that disturbances in this wavelength range were not serious when the angle of grazing incidence was 5.5°. As the grazing angle increases, the effect of absorption edges on the reflectance becomes more pronounced [46]. The complex index of refraction of many elements and compounds has been determined by Lukirskii et al. [47] from reflectance measurements at wavelengths between 23.6 and 113 Å measured as a function of the angle of incidence.

For radiation shorter than about 1000 Å, platinum is generally the most useful and efficient reflector, especially at normal incidence. Unlike aluminum, platinum shows very little change in reflectance after exposure to air [48,49]. However, the actual magnitude of the reflectance of most materials including platinum depends critically on the method of producing the reflecting surface. Most reflecting films are produced by vacuum evaporation either from resistance-heated filaments or by induction heating. For highest quality reflecting films, it is generally necessary to deposit the films as rapidly as possible and at a high vacuum, typically 5 × 10⁻⁶ torr or better. To obtain a smooth surface for maximum reflectance, the deposited film is usually kept as thin as possible while still being opaque to the incident radiation. For platinum, the optimum thickness is about 100 Å as measured by Jacobus et al. [49]. The reflectance was down about 10 percent for a 500 Å thick film.

Gold appears to be a second choice for a suitable reflector for wavelengths shorter than 1000 Å. Like platinum, gold is very stable on exposure to air and provides the highest reflectance for film thicknesses of approximately 100 Å [50]. Figure 2.22 shows the reflectance of gold and
platinum between 400 and 1600 Å. The increase in reflectance around 584 Å appears to be characteristic of both gold and platinum. The results for gold shown in Fig. 2.22 agree well with those of Canfield et al. [50] (see Chapter 9). The published data on platinum [49,51,52] tend to lie both above and below those shown in Fig. 2.22, emphasizing the variation which can be obtained under the different conditions of production.

In the past aluminum has been the most commonly used material to coat mirrors and gratings. However, Hass et al. [53] have shown that although aluminum is an excellent material for use above 2000 Å, it has a very serious aging effect at shorter wavelengths. This aging effect has been traced to the formation of an oxide layer. As the thickness of the aluminum oxide layer builds up, the reflectance continually decreases. When freshly deposited aluminum is irradiated with ultraviolet light, there is a speed up in the oxide formation. Figure 2.23 illustrates this aging effect, showing the reflectance of aluminum measured in vacuum immediately after deposition and then remeasured after an exposure of one hour, one day, and one month to air [54]. Hass and Tousey [52] showed that the excellent reflectance of aluminum could be maintained by preventing the growth of oxide layers. To prevent oxidation, they overcoated freshly deposited aluminum with a thin protective layer of magnesium fluoride. The results were quite dramatic; the reflectance between 1200 and 2000 Å was about 80 per cent and showed no decrease on exposure to air. More recently, Canfield et al. [55] have reported results of extended tests on MgF₂ overcoatings. They have observed no aging effects nor any loss in reflectance after irradiation with 1-MeV electrons and 5-MeV protons. The optimum thickness of MgF₂ coating depends upon the wavelength. Figure 2.24 shows the reflectance of Al + MgF₂ as a function of the thickness of the MgF₂ [56]. A film thickness of approximately 220 Å is a fairly good compromise for radiation between 1025 and 1600 Å. The use of LiF overcoatings has the effect of maintaining the reflectance of aluminum at shorter wavelengths. This is because the transmission limit of LiF is slightly lower than that of MgF₂. The results obtained by Hunter [56] are shown in Fig. 2.25. The optimum thickness of LiF should be about 170 Å to maintain high reflectance at the shorter wavelengths. LiF, however, is rather hydroscopic and has a reduced transmittance on exposure to a humid atmosphere. Hunter prevented this by overcoating the Al + LiF combination with a very thin protective layer of MgF₂ about 15 Å thick.

The reflectances of many materials have been measured in the vacuum uv [57,60], and an excellent review has been given by Madden on the techniques for preparing and measuring reflective coatings [61]. At present, the most efficient reflective coatings for gratings in general use appear to be Al + MgF₂ for the wavelength range 2000 to 1000 Å and platinum for wavelengths below 1000 Å.
The use of a focused laser beam of high instantaneous power is currently being used by the author to produce reflective coatings of refractory materials of high density and high melting points. Thin films of a few hundred angstroms thick can be deposited in approximately 100 μsec and under conditions of ultrahigh vacuum. This technique for producing high reflective coatings appears to be very promising. Preliminary measurements on thin iridium films give a reflectance curve that is slightly greater at all wavelengths than the platinum curve shown in Fig. 2.22.

REFERENCES

Vacuum Spectrographs and Monochromators

The dispersing element of a spectrograph is normally a prism or a diffraction grating. However, although spectrographs have been made using lithium fluoride prisms enabling the spectrum to be studied down to about 1100 Å, we will confine the discussion of this chapter to diffraction grating vacuum spectrographs and monochromators. The advantages of the grating spectrograph are wider wavelength coverage, higher resolving power, less scattered light, nearly linear dispersion (in the case of normal incidence), and precise wavelength measurements. Moreover, few if any vacuum prism spectrographs are available commercially, whereas numerous grating spectrographs and monochromators are manufactured.

3.1 SPECTROGRAPHS

The basic principle of the concave grating spectrograph lies in the focusing properties of the concave diffraction grating. As discussed in Chapter 2, these properties are such that the diffracted images of the source are sharply focused on a circle, called the Rowland circle, of a diameter equal to the radius of curvature of the grating, provided the surface of the grating is tangential to the Rowland circle, the ruled lines are at right angles to the Rowland plane, and the illuminated entrance slit is on the Rowland circle and parallel to the ruled lines of the grating. These conditions are essential in providing the highest possible resolving power. Figure 3.1 shows the basic optical layout of a spectrograph. The absolute values of the wavelengths can be determined from the following equations:

GRATING EQUATION, \[ \pm m\lambda = d (\sin \alpha + \sin \beta); \]  \hspace{1cm} (3.1)

RECIPROCAL DISPERSION, \[ \frac{d\lambda}{d\lambda} = \frac{d}{mR} \cos \beta, \]  \hspace{1cm} (3.2)
where the positive sign in (3.1) applies to the "inside orders," and the negative sign to the "outside orders." The signs of $\alpha$ and $\beta$ are opposite when they lie on different sides of the grating normal.

There are two basic types of spectrograph, the normal incidence spectrograph suitable for studies from 300 to 2000 Å and the grazing incidence spectrograph essential for studies below 300 Å. Spectrographs are generally referred to by the size of grating and the type of mount. Thus an instrument using a grating with a radius of curvature $R$ of 1 m and with an angle of incidence close to zero is known as a 1 meter normal incidence spectrograph.

**Normal Incidence Mount.** When the angle of incidence $\alpha$ is less than approximately $10^\circ$, the radiation is considered to be directed at normal incidence to the grating. For $\alpha$ less than $10^\circ$, there is very little astigmatism and essentially no change in the reflectance, hence efficiency, of the grating. The actual angle of incidence, then, is dictated by the physical problem of mounting the photographic plate holder and light source.

The inside spectrum is normally used either in its first or higher orders since there is actually a loss in resolving power as $\beta$ increases (see Chapter 2, Sect. 2.3). There is also a considerable saving in space when the inside spectrum is used.

Figure 3.2 shows a typical normal incidence spectrograph. The positions of the first order wavelengths for a 1200 line per mm grating are indicated when the angle of incidence $\alpha = 10^\circ$. For these parameters, the astigmatism at 500 Å increases the image length of the slit by approximately $\lambda$.

![Fig. 3.1 Optical layout of a basic spectrograph.](image)

![Fig. 3.2 Normal incidence spectrograph.](image)

![Grating (radius $R$)](image)

Fig. 3.1 Optical layout of a basic spectrograph.

The inside spectrum is normally used either in its first or higher orders since there is actually a loss in resolving power as $\beta$ increases (see Chapter 2, Sect. 2.3). There is also a considerable saving in space when the inside spectrum is used.

Eagle Mount [1,2.] In the Eagle mount, $\alpha \sim \beta$, and for this reason, the original Eagle design required the slit and plateholder to be located at some distance above and below the Rowland plane ("off-plane" mount) in order to avoid physical interference with each other. The angle of incidence is not restricted. However, when $\alpha$ is small, the normal incidence
mount approximates to that of an “in-plane” Eagle mount with the spectrum viewed close to the entrance slit.

It would appear that the “off-plane” Eagle mount would adversely affect the resolving power of the spectrograph since the slit and plateholder are no longer on the Rowland circle. It has been shown by Wilkinson [3] and Namioka [4], however, that high resolution can still be obtained when the angular deviation of slit and plateholder is small. Presumably, then, for a fixed separation between the slit and plateholder, the larger the instrument, the less defocussing obtained. With their 21 ft vacuum spectrograph, the slit and plateholder subtended an angle of about 2° at the grating. Resolving powers of 300,000 in the fourth order were obtained. To obtain high resolution, Wilkinson [3] has shown that the entrance slit must be rotated slightly. Since rotation of the slit turret is a standard procedure to produce optimum results in any spectrograph, the proper angle of inclination of the slit will be obtained automatically. Namioka [4] has shown that the amount of rotation ϕ is given analytically by

$$\phi = \frac{z_0}{R} \tan \alpha \sec \alpha,$$  (3.3)

where $z_0$ is the displacement of the slit center above the Rowland circle, and α is the angle of incidence for the rays from the center of the slit. The exact grating equation (2.15) must now be used to determine the various wavelengths; that is,

$$\frac{m\lambda}{d} = \left(1 + \frac{z_0^2}{R^2 \cos^2 \alpha}\right)^{1/2} (\sin \alpha + \sin \beta).$$  (3.4)

The optimum width of a grating in the Eagle mount is given by [4]

$$W_{opt} = 2.12 \left(\frac{4z^3 \lambda}{\cos^2 \alpha} + \frac{2R^2 \lambda \cos \alpha \cos \beta}{\sin^2 \alpha \cos \beta + \sin \alpha \cos \beta \cos \alpha}\right)^{1/4} - \frac{2z_0^2}{\cos^2 \alpha}.$$  (3.5)

Figure 3.3 shows a typical arrangement of optical parts in the off-plane Eagle mount. The light source is positioned at the entrance slit S, which is at the center of the plateholder. The vertical separation between S and the plateholder is 2z0. When the grating is at the position G, the central image falls at the center of the plateholder, allowing the instrument to be focused with ease. To change the wavelength interval under study, G is rotated about a vertical axis until the wavelength region of interest falls on the plateholder. Then G is moved along the track GS to the position G'. That is, the Rowland circle has been rotated about the fixed point S and the grating moved such that $\alpha = \beta$. The plateholder is then rotated about S until it is back on the Rowland circle. For work at high orders of the spectrum, the adjustments again can be made in the visible. For example, the fourth order of the 1200 to 1500 Å region can be focused in air by using the mercury 5461 Å line. Final adjustments are made under vacuum by using appropriate sharp emission lines.

The largest concave grating vacuum spectrograph is the 35-foot (10.7 m) in-plane Eagle mount. This instrument was first designed and built by Douglas [5,6]. It is now built commercially by the Jarrell-Ash Company (see Fig. 3.4). The ruled width of the 1200 line per mm grating is 7 in. This gives a reciprocal dispersion of 0.8 Å per mm in the first order while a resolution of 200,000 is reported at 1800 Å. The commercial instrument has a wavelength range of approximately 400 to 8000 Å.

The spectrograph consists of a vacuum tank 50 in. in diameter on one end and tapering along its 40 foot length to 36 in. The vacuum tank, unlike those in most large spectrographs, does not support the structural optical assembly. Instead, the optics are mounted on concrete piers (see Fig. 3.5) that, in turn, are independent of the building within which the instrument is housed. Large stainless steel posts pass through the vacuum wall by means of vacuum-tight flexible couplings. The optics are thus independent of any distortion introduced into the vacuum vessel when the unit is pumped as well as any vibration from pumps or external sources.
of the ruled lines of the grating. The grating equation, according to Bragg's law, is

\[ m\lambda = 2d \sin \theta, \]

where \( \theta \) is the grazing angle, \( m \) is the order number, and \( d \) is the distance between reflecting planes of the crystal. Thus the maximum wavelength diffracted by a crystal is \( \lambda = 2d \).

Synthetic crystals of potassium acid phthalate have been made with lattice spacings \( d \sim 13 \text{ Å} \) and have proved useful for diffraction of wavelengths as long as 25 Å [10]. Recently, new crystals with larger interplaner spacings have been produced that are suitable for soft x-ray spectroscopy in the region 25 to 100 Å [11a].

To obtain the highest resolution in such large spectrographs, the grating temperature must be held constant to within 0.05°C.

Grazing Incidence Mount. The decrease in reflectance with decreasing wavelength of all grating materials necessitates the use of grazing incidence spectrographs for wavelengths below 200 to 300 Å. By making use of the total reflection experienced at extreme grazing incidence, Tyrén [8], using an angle of incidence of 89°, observed wavelengths as short as 12 Å. Kirkpatrick [8], using a grazing angle of 51° of arc, photographed x-ray spectral lines down to 7 Å. The theoretical minimum wavelength for this grazing angle on glass is 5.6 Å (see (2.48)). As discussed in Chapter 2, the theoretical wavelength cutoff is lowered by using a reflecting surface of gold or platinum. Gabriel et al. [9] have recorded the shortest wavelength yet, namely 4.7 Å, by using a platinized grating at a grazing angle of 2°. For a platinum coating, the critical wavelength (in angstroms) is approximately equal to 2.64 times the grazing angle in degrees. Therefore, at a grazing angle of 2°, the wavelength cutoff should be approximately 5.3 Å.

The optical arrangement for a grazing incidence mount is shown in Fig. 3.6.

To disperse shorter wavelengths the diffraction grating must be replaced by a crystal. The regular lattice spacing in the crystal now takes the place of the ruled lines of the grating. These crystals are organic esters such as octadecyl hydrogen maleate (2d \sim 64 Å), dioctadecyl adipate (2d \sim 94 Å), and octadecyl hydrogen succinate (2d \sim 97 Å). The cleavage of these crystals is similar to mica. Although the crystals are relatively soft, they have sufficient mechanical strength to be handled without difficulty and can be easily bent to provide a degree of focusing. X-ray wavelengths are usually measured in x-ray units (xu) based on the lattice spacing of NaCl. At present, these x-ray units are 0.202 per cent smaller than the angstrom (10^{-8} cm). Although the angular dispersion increases at grazing incidence, the resolving power actually decreases. This can be seen from the fact that the resolving power is proportional to \( W_{\text{opt}} \), the optimum width of the grating, which, as shown in Table 2.2, rapidly decreases as the angle of incidence increases. For small gratings, say of 1 m radius, the optimum width and hence resolution is down by almost a factor of 10 at 88° compared with normal incidence. However, for larger instruments, since there is a practical limit to the width a grating can be ruled, the discrepancy in resolution becomes less.

Comparing the performance of a 6.8 m grazing incidence spectrograph (\( \alpha = 82° \)) with a 6.8 m Eagle mount, where both use 1200 line per mm
gratings in the first order at 1000 Å, we see that the reciprocal dispersion in the grazing incidence mount is 0.64 Å per mm, while it is 1.28 Å per mm for the Eagle mount; a factor of 2 better at grazing incidence. The optimum width of the grating at 82° and 1000 Å is about 11 cm, whereas in the Eagle mount it is about 50 cm. However, the largest gratings available at present have ruled widths of only 20 cm, and this, then, limits the resolution obtained by the Eagle mount. Nevertheless, the resolution is a factor of 2 better than in the grazing incidence spectrograph and essentially compensates for the smaller dispersion. It is, of course, assumed that the total grating area is illuminated in both cases and that the slit widths are infinitely narrow. Thus it would appear that the performances of the two instruments are comparable in the first order.

The actual choice of instrument for work above 500 Å depends on many factors. For example, the grazing incidence spectrograph discussed above requires a volume twice that of the off-plane Eagle mount. Of course, the overall length of the grazing incidence spectrograph is about 12 ft compared to 21 ft for the Eagle mount. This may be an important factor where laboratory space is at a premium! With the given volume the grazing incidence spectrograph is limited in spectral range making it difficult or impossible to work in higher spectral orders. With the Eagle mount, on the other hand, it is relatively simple to study the fourth or fifth order spectrum. Finally, the astigmatism is very much greater at grazing incidence requiring much greater precision and care in the focusing of the spectrograph. Although there is an increase in the overall dispersion, it would appear that the major advantage of the grazing incidence mount is in enabling the spectrum to be studied from 400 Å down to the soft x-rays.

**Echelle Mount.** Echelle in French means “ladder” or “pair of steps.” The diminutive form is *echelle*, while a rung of a ladder is *echelon*. These three words have all been used to describe regular steps in a reflecting material causing the diffraction of light.

The earliest form was Michelson’s [12,13] *echelon*, which consisted of a series of optically flat glass plates of identical widths stacked together, but offset slightly to form a “ladder.” Owing to the difficulties involved in preparing identical plates with the required precision, the echelon seldom has more than 40 plates and is extremely expensive to construct.

R. W. Wood [14] originated the shaped grooves of a diffraction grating (plane or concave) and called the grating an *echelle*. The grooves were shaped to increase the energy diffracted in a given order. This is the basis for the present day “blazed” gratings.

G. R. Harrison [15–17] designed an instrument with characteristics intermediate between the echelle and the echelon and called it an *echelle* grating. The echelle is essentially a ruled echelon. The precision of the grooves must approach that of the echelon. The echelle generally has more steps than the echelon, but fewer than the echelle. Resolving powers of 1 million have not yet been achieved with a concave diffraction grating. At 1000 Å this represents a resolution of 0.001 Å. To obtain resolving powers as high or even higher, it is necessary to use the echelle mount. This consists of a plane diffraction grating with carefully shaped grooves used in the Eagle-type mount with α = β ≈ 90°, and with a concave mirror to focus the image. The same mirror may also be used to render the incident light parallel since the echelle grating must be used with parallel light. High resolution is obtained by working in high spectral orders, typically m ~ 1000. This means that some form of predisperser is necessary to separate overlapping orders.

Where only a narrow spectral range is under study, a predisperser can be used to isolate the region of interest. Then an echelle must be chosen with a free spectral range equal to the band pass given by the predisperser. The free spectral range $F$ is given by

$$F = \frac{\lambda}{m},$$

where $m\lambda$ ~ twice the groove separation, as proved below (see 3.10). Thus for 1000 Å in its 1000th order, the free spectral range is only 1 Å. The band pass of the predisperser must therefore be \( \leq 1 \) Å. The echelle in this case should have only 20 grooves per mm. The predisperser aids also in reducing scattered light.

On the other hand, when the echelle is crossed with a concave grating (i.e., the rulings of each grating are perpendicular to each other), separation of overlapping orders can be achieved with the additional advantage that a very large spectral range can be covered in a very small area. This is because the echelle produces the primary spectrum in one plane, whereas the concave grating separates the overlapping orders in a plane at right angles to that of the primary spectrum.

With the increased number of reflections necessary, the echelle mount has been used only above 1000 Å. Figure 3.7 shows a typical echelle spectrograph with a concave grating predisperser to separate higher orders.

Diffraction gratings normally achieve high resolution and dispersion by using a large number of grooves and working in moderately high orders, $m \sim 4$ to 20. From the grating equation $m\lambda = d(\sin \alpha + \sin \beta)$ and from the resolving power $R = (m/d)W_{\text{opt}}$, we obtain

$$R = \frac{W_{\text{opt}}}{\lambda} (\sin \alpha + \sin \beta).$$

(3.8)
For a given \( \lambda \) the resolving power increases when \( \alpha \) and \( \beta \) are on the same side of the grating normal, whereas \( W_{\text{opt}} \) is independent of the sign of \( \beta \). Further, \( R \) increases to a maximum as \( \alpha \) and \( \beta \) increase, then decreases to zero at \( \alpha = 90^\circ \). If we let \( \alpha = \beta \) in (3.8) and differentiate \( R \) with respect to \( \beta \), we find that \( R \) is a maximum at \( \alpha = \beta = 54^\circ45' \). The dispersion, on the other hand, is proportional to \( 1/\cos \beta \) and therefore continues to increase as \( \beta \) increases. Dispersion is the more important quantity when resolution is limited by the practical slit width necessary to record a spectrum.

For a plane diffraction grating there is no optimum width. Thus from (3.8) the maximum resolving power is achieved when \( \alpha = \beta \approx 90^\circ \), namely,

\[
R_{\text{max}} = 2 \frac{W}{\lambda}. \tag{3.9}
\]

In this form we see that the maximum resolving power for a given wavelength is simply proportional to the width of the grating illuminated (and quite independent of the number of grooves).

For a grating 5 cm wide a resolving power of \( 10^6 \) should be obtained at 1000 Å. This suggests that it should be possible to rule a few properly shaped grooves so that most of the diffracted energy is returned into the desired order. From Chapter 2 we saw that the maximum energy went into the wavelength diffracted in the same direction as the specularly reflected beam from the surface of the facets, that is, for \( 2\theta = (\alpha + \beta) \). Under the conditions for maximum resolution this becomes \( \theta = \alpha = \beta \approx 90^\circ \). Figure 3.8 illustrates the echelle grating and shows its similarity to Michelson's reflecting echelon. In fact, the principle of the echelle is identical to that of the echelon. The difference lies only in the number of steps and in their production. With the parameters in Fig. 3.8, the grating equation \( m\lambda = d(\sin \alpha + \sin \beta) \) of the echelle becomes

\[
m\lambda = 2t - s\theta, \tag{3.10}
\]

which is the equation for the echelon, while the angular dispersion is given by

\[
\frac{d\theta}{d\lambda} = \frac{m}{s}. \tag{3.11}
\]

The ruling conditions for an echelle grating are very severe. The reflecting surfaces \( s \) must be optically flat and the separation \( t \) between steps must be held to within \( \frac{1}{1000} \lambda \). The inaccuracies in spacing of the grooves greatly decrease the actual resolving power of a grating at high angles of incidence. The ruling of the gratings are controlled by interferometric methods to achieve such perfection.

Further dispersion can be achieved by using two echelles in tandem. Although this has been successful in the visible [17,18], every extra reflection in the vacuum ultraviolet region produces a large decrease in intensity.

The echelle grating and the echelon appear to be the only instruments capable of producing resolving powers in excess of \( 10^6 \) in the vacuum uv.
Separation of the overlapping orders can be achieved by crossing the echelle and echelon with a concave diffraction grating.

**Ebert-Fastie Mount.** Another plane grating mount (similar to the echelle mount) with three reflections was designed by Ebert [19] in 1889 and rediscovered and modified by Fastie [20,21] in 1952. Although it has been used mainly for the near ultraviolet and visible region of the spectrum, it should be useful down to 1100 Å when the reflecting components are coated with Al-MgF₂ films.

![Ebert-Fastie Mount](image)

The optical system is shown in Fig. 3.9. Light from the entrance slit \( S_1 \) is rendered parallel by the spherical concave mirror \( M \) and is reflected onto the plane grating \( G \). The diffracted rays are then refocused by \( M \) onto the photographic plate \( P \).

In its original form, as described by Ebert, the center of the grating lay in line with \( S_1 \) and \( P \). In this position, however, only a small spectral range was in focus at the plate, this portion being situated at the same distance as \( S_1 \) from \( G \), but on the opposite side. The focal plane is actually curved. If \( G \) is located at a distance from \( M \) of about 0.8 times that of \( S_1 \), the focal plane of the spectrum will lie in a straight line.

Instead of the single mirror, two smaller ones could be used as in the Czerny-Turner system [22]. Fastie has shown that a very high resolution can be achieved with this mount.

**Miscellaneous Mounts.** All the standard optical mountings of concave gratings, such as the Rowland, Paschen-Runge, and Abney mounts, used in the visible and infrared region are all applicable to the vacuum ultraviolet. However, they simply describe various mechanisms for changing the spectral region to be photographed; the spectrographs themselves are still based on the Rowland circle.

A somewhat different optical arrangement of the concave grating is its use with parallel light. This type of mounting was first described in 1896 by Wadsworth [23], who found that stigmatic images could be obtained from a concave grating when viewed near the normal to the grating. From Sect. 2.1, it is shown that the images are formed at a distance \( r' \) from the grating given by

\[
r' = \frac{R}{1 + \cos \alpha}
\]

This is the equation of a parabola. For a given \( \alpha \), a stigmatic spectrum is generated in the vicinity of the normal, where the photographic plate has an approximate curvature of a circle with radius \( R/(1 + \cos \alpha) \). In changing the spectral range, \( \alpha \) is varied; thus the curvature of the plate holder must be changed. Though truly stigmatic images are obtained only on the normal, for most purposes a range of several hundred angstroms can be obtained at one setting.

Figure 3.10 shows the arrangement of the Wadsworth mount as used by Meggers and Burns [24]. The spectral range is varied by rotating the grating \( G \) about its center. The plateholder is attached to and free to move along the rod \( GP \), which is also the normal to \( G \).
The Wadsworth mount has been used in the construction of a simple spectroheliograph for photographing a stigmatic image of the sun at the He II 304 Å line [25].

3.2 MONOCHROMATORS

As in the vacuum spectrograph, the use of a concave grating as the dispersing device requires that the optical components of the monochromator always lie on the Rowland circle to produce perfect focus. The optical parts in such a case are the entrance and exit slits and the concave grating. To maintain perfect focus and still achieve monochromatic radiation from the exit slit, either one element must move along the Rowland circle, or else the Rowland circle must rotate about one of the elements. The other two elements are moved appropriately so that they stay on the circle. Several mountings described below achieve monochromatic action and still maintain focus.

There are many situations in which a monochromator with perfect focus is not so important as such overriding considerations as fixed exit or entrance slits, size, cost, an undeviating exit beam, and others. Thus there is probably no ideal monochromator. The selection of a design is dictated generally by the application. Certainly the simplest and most straightforward forward monochromator would be one in which the grating was simply rotated about its center. If the instrument initially had the optical elements on the Rowland circle for the wavelength in the middle of the spectral range under study, the defocusing would be minimized.

Some of the more important monochromator designs are discussed below. All of these designs attempt to counteract any defocusing of the instrument.

Rowland Circle Mounting

Grazing Incidence. In the grazing incidence mounting any deviation of the slits or the grating from the Rowland circle results in a very large degree of defocusing. Thus monochromatic action at grazing incidence is confined to moving one or more of the optical elements along the Rowland circle.

One of the earliest vacuum monochromators and the first grazing incidence monochromator was constructed by Baker in 1938 [26]. In his instrument both slits were fixed and the grating was constrained to move on the Rowland circle. In this geometry both the angles of incidence and diffraction change, but the angle subtended by the two slits at the center of the grating remains constant; that is, \((\alpha - \beta) = \text{constant}\) (remembering that the sign of \(\beta\) is opposite to that of \(\alpha\) if it lies on the opposite side of the grating normal). The angular dispersion in this case is very nearly constant over a limited range of \(\beta\) and is given by

\[
\frac{d\lambda}{d\beta} = 2d \cos \varphi \cos (\varphi + \beta),
\]

where \(\varphi\) is a constant equal to \(\frac{1}{4}(\alpha - \beta)\). If fixed exit and entrance slits are required, then this is the only appropriate mounting.

Piove et al. [27] used a movable exit slit and were thereby able to make use of the increasing dispersion at shorter wavelengths. Landon [28] describes a similar instrument, but his scanning mechanism allows a linear wavelength readout. Figure 3.11 illustrates his scanning system. A rigid rod \(OG\) is attached to the grating \(G\), while a movable rod \(OS_2\) is pivoted at \(O\), the center of the Rowland circle, and fixed to the exit slit \(S_2\). A precision screw \(S\) is aligned to bisect the isosceles triangle \(OGS_2\), and a rigid rod \(NS_2\) is fixed to the precision nut \(N\) and to the exit slit. Two supporting rods are also fixed to \(N\) and pivoted at \(A\) and \(B\), the midpoints of \(OS_2\) and \(OG\), respectively. Rotation of the screw constrains the exit slit to move along the Rowland circle. The length of the screw \(ON\) is always equal to \(OG \sin \beta\). However, since the angle of incidence is constant, the wavelength is also proportional to \(\sin \beta\). Thus the length of \(ON\) is proportional to the wavelength, and a linear wavelength scale is obtained that can be read out on a counter.

A novel method of moving the exit slit has been used by Hinteregger [29,30] and associates in a rocket monochromator for scanning the solar spectrum from about 1300 to 60 Å. Their method consisted of cutting several exit slits in an endless steel band that was constrained to move along a segment of the Rowland circle. The radiation was detected by a
fixed electron multiplier with a cathode sufficiently large to receive the sweeping exit beam. A schematic of the monochromator is shown in Fig. 3.12.

The principal use for this type of monochromator is, of course, in the study of spectra from various sources, not in its ability to provide monochromatic radiation for other research projects.

Another instrument utilizing the moving exit slit has been produced by the McPherson Instrument Corp. [31]. In this case the exit slit is mounted on a turntable that moves along a precision machined track coinciding with the Rowland circle. The exit and entrance slits are readily interchangeable. For the movable slit to be accessible outside the vacuum housing, the slit is connected to the vacuum system through a series of telescoping tubes and O-ring seals. Figure 3.13 is a photograph of a 2 m McPherson grazing incidence monochromator shown with the exit slit mounted on the track and the telescoping tubes.

A grazing incidence monochromator in which the Rowland circle is pivoted about the exit slit keeping the angle of incidence constant has been constructed by Salle and Vodar [32]. Figure 3.14 shows a typical arrangement of their scanning mechanism. The grating $G$ is rigidly fixed to the entrance slit $S_1$ at a fixed angle of incidence $\alpha$. The length of the rod $GS_1$ is, therefore, $R \cos \alpha$. Both $G$ and $S_1$ will subtend a constant angle of $(90 - \alpha)$ at $S_2$. The grating is mounted on a turntable constrained to move along the track $GS_2$, and $S_1$ is constrained to move along the track $S_2S_3$. Thus any movement of $S_1$ along $S_2S_3$ will result in a larger displacement of the grating along $S_3G$ coupled simultaneously with a rotation of the grating relative to $S_2G$. Because $GS_1$ and $\alpha$ are constants, it is easily seen that any movement of $S_1$ along $S_2S_3$ still maintains $S_1$ and $G$ on the Rowland circle.

This mounting is particularly advantageous in providing a constant angle of incidence and, at the same time, a fixed exit slit and a fixed...
Fig. 3.14 Vodar grazing incidence monochromator. Scanning mechanism maintains the optical elements on the Rowland circle and maintains a constant angle of incidence. The Rowland circle is pivoted about the exit slit $S_2$.

direction of the emergent beam. Furthermore, the dispersion increases rapidly to shorter wavelengths.

**Normal Incidence.** All the techniques described above to create a grazing incidence monochromator can also be applied to normal incidence monochromators. To obtain precise focusing as the spectrum is scanned, the optical elements must lie on the Rowland circle. Actually, there is one exception to this rule when $\alpha = \beta$. This is the condition for the off-plane Eagle monochromator discussed below.

As in the grazing incidence case, if fixed exit and entrance slits are required, the grating must slide along the Rowland circle. This was the technique adopted by Tousey et al. [33] and by Fujikawa and Ito [34] in their *radius mounting* [35]. The grating was mounted at the end of an arm pivoted about the center of the Rowland circle. The arm coincided with the normal to the grating. The motion of the grating causes the exit beam to change direction unless the exit beam is sufficiently baffled. This, of course, decreases the effective width of the grating available for dispersion and decreases the intensity of the exit beam.

To avoid the changing aspect of the grating as seen from the axes of both the light source and the detector Clarke and Garton [36] moved both the exit and entrance slits on tracks directed from the respective slits to the center of the grating. The grating was allowed to rotate about its center.

Fig. 3.15 Scanning mechanism for McPherson 2 m combination normal incidence spectrograph and scanning monochromator.

The grating and the two slits were linked by arms pivoted at the center of the Rowland circle $O$. Any displacement of $O$ caused the grating to rotate and the slits to slide along their tracks, remaining automatically on the Rowland circle since the arms were radii of the Rowland circle.

McPherson [31] used a fixed entrance slit and allowed the grating and exit slit (or plateholder) to move in his 2-m combination vacuum spectrograph and scanning monochromator. Figure 3.15 shows the scanning mechanism. In the initial position, the grating is at $G_0$ with the light from the fixed entrance slit $S$ at an angle of incidence of 8°15', while the exit slit $X$ is at the opposite end of a diameter passing through $G_0$. The plateholder is tangential to the diameter at $X$. Both the grating and $X$ are rigidly attached to arms pivoted at $O$. Then by displacing $O$ along the arc of a circle, with center $S$, $G_0$ and $X$ slide along tracks $G_0S$ and $G_0X$, respectively. In this arrangement there is a very slight change in the direction of the exit beam.

**Off-Rowland Circle Mounting**

The following monochromators have fixed exit and entrance slits, the scanning action being achieved by a simple rotation or by a simultaneous rotation and translation of the grating.
To find the linear motion $x$ of the grating it is noted that in Fig. 3.16, to a good approximation, let $\theta_0$ be the angle of incidence in the initial position. Then, if the change in the angle subtended by the slits at the grating is small over the spectral range scanned, (3.16) becomes, for $s = s'$:

$$R \cos \alpha + s = R \cos \beta - s'.$$  

Solving for $s'$, (3.14) becomes

$$s' = s \left[ 1 + \frac{s}{R} \left( \frac{\cos \alpha + \cos \beta}{\cos \alpha \cos \beta} \right) \right]^{-1}. \quad (3.15)$$

When

$$\frac{s}{R} \left( \frac{\cos \alpha + \cos \beta}{\cos \alpha \cos \beta} \right) \ll 1,$$

then $s' \approx s$,

which is approximately the case for near normal incidence. Referring to Fig. 3.16, we see that if the grating $G_0$, entrance slit $N_0$, and exit slit $X_0$ are initially on the Rowland circle such that the central image falls on $X_0$, then for a rotation of the grating through an angle $\delta$ and a linear motion through a distance $x$, the optical elements should now be at $G$, $N$, and $X$. Thus we see that the entrance slit lies a distance $s$ outside the new Rowland circle, whereas the exit slit lies a distance $s'$ inside the circle. The distance $s$ is found as follows.

By construction, we have $GN_0 = GX_0$; that is

$$R \cos \alpha + s = R \cos \beta - s'.$$  

Let $\alpha_0$ be the angle of incidence in the initial position. Then, if the change in the angle subtended by the slits at the grating is small over the spectral range scanned, (3.16) becomes, for $s = s'$,

$$s = R \sin \alpha_0 \sin \delta.$$  

(3.17)

To find the linear motion $x$ of the grating it is noted that in Fig. 3.16, to a good approximation,

$$s = R \cos (\alpha - \delta) - R \cos \alpha - x \cos (\alpha - \delta)$$  

and

$$s' = -R \cos (\alpha - \delta) + R \cos \beta + x \cos (\alpha - \delta),$$  

where $R = G_0P_0 = GP$ and $R - x = GP_0$. Now if $s = s'$, then by equating (3.18) and (3.19) and noting that $\delta = \frac{1}{2}(\alpha + \beta)$, we obtain

$$x = R(1 - \cos \delta).$$  

(3.20)

A more rigorous analysis using similar triangles $G_0GN_0$ and $G_0GX_0$ gives

$$x = \frac{R}{2} \sin^2 \delta - \frac{R}{2} \tan^2 \alpha_0.$$  

(3.21)

However, for $\delta$ small, (3.20) is sufficiently accurate. Moreover, it is the precise equation obtained if the Rowland circle is pivoted about $P_0$, the point of intersection of the grating normal with the Rowland circle in its initial position. The use of (3.20) as the focusing condition leads to a
simple scanning mechanism. For example, if the grating is fixed to an arm lying along its normal and pivoted at the center of the Rowland circle \(O\) (Fig. 3.16), and if the arm \(OP_e\) is also pivoted at \(O\) and at the fixed point \(P_e\), then a simple movement of \(O\) will pivot the Rowland circle about \(P_e\). The grating, of course, is constrained to move along the track \(G_0P_e\). This type of mounting and scanning mechanism has been described and analyzed by Yodar [37] and Robin [38] and is produced commercially by the Jarrell-Ash Corporation*. A more rigorous analysis has been given by Pouey and Romand [42].

An empirical approach has been used by McPherson [39]. In his instrument† a lever arm of suitable length greater than \(R/2\), but less than \(R\), is rigidly attached to the grating mount. The arm coincides with the grating normal in the initial position. The end of the lever arm is constrained to follow a properly shaped cam. The curvature of the cam is determined empirically by focusing various wavelengths at the exit slit in the range 0-6000 Å and noting the position of the cam follower. These positions are found to lie on a smooth curve. The cam is then machined to this curvature. One advantage of this procedure is that the curvature of the cam automatically compensates for any errors in the original positioning of the grating. With slit widths of 10 μ and a 1 m, 600 line per mm grating, a resolution of approximately 0.23 Å has been obtained over the range 0-6000 Å.

**Eagle off-plane Mounting.** This can be considered as a special case of the mounting described in the preceding section, for, when the entrance and exit slits coincide the bisector becomes simply the line joining the slits to the grating. In the off-plane Eagle mounting the entrance and exit slits are placed symmetrically above and below the Rowland plane, so that the distance \(z_0\) between the center of the slit and the Rowland plane is the same for both slits. The angle of incidence \(α_0\) for the rays from the center of the entrance slit is equal to the angle of diffraction \(β_0\) for the rays that originate at the center of the entrance slit and are diffracted from the center of the grating rulings. Both \(α_0\) and \(β_0\) are measured on the Rowland plane.

To scan the spectrum the grating is rotated and moved in the direction of the fixed slits, whereas the Rowland circle pivots about the slits and no defocusing of the normal type occurs; that is, no element is forced off the circle as the spectrum is scanned. The question, however, is to what extent the monochromator already is defocused by locating the slits above and below the Rowland plane. As was discussed earlier in Sect. 3.1, Wilkinson [3] and Namioka [4] have shown that it does not defocus a spectrograph appreciably when a large radius grating is used, provided the entrance slit is slightly rotated with respect to the rulings of the grating. Namioka has shown the following conditions to hold [43].

\[
\text{SLIT ROTATION } \varphi = \frac{z_0 \tan α \sec α}{R}; \tag{3.22}
\]

\[
\text{WAVELENGTH } \lambda = \frac{2d}{m} \left(1 + \frac{z_0^2}{R^2 \cos^2 α}\right)^{-\frac{1}{2}}; \tag{3.23}
\]

\[
\text{OPTIMUM WIDTH OF GRATING } W_{\text{opt}} = 2.12 \left(\frac{4z_0^4}{\cos^4 α} + \frac{λR^3 \cot α}{\sin α}\right)^{\frac{1}{2}} - \frac{2z_0^2}{\cos^2 α}. \tag{3.24}
\]

The symbols have been defined in Section 3.1. Typical values of \(λ\), \(φ\), and \(W_{\text{opt}}\) are approximately 591 Å, 2.5' of arc, and 37 cm, respectively, for a 3 m grating with 1200 lines per mm used in the first order at an angle of incidence equal to 2° and with \(z_0 = 6\) cm. It can be seen that the optimum width of the grating is much larger than that currently available on the market.

The scanning mechanism shown in Fig. 3.3 for the off-plane Eagle spectrograph is similar to that of McPherson's shown in Fig. 3.15. For the slits to be fixed and the grating constrained to move along tracks joining the grating and the slits, the center of the Rowland circle must move along an arc of a circle with center \(S\) and radius equal to that of the Rowland circle. The center \(O\) need not physically be moved along the arc, and any link mechanism which effects a virtual displacement of \(O\) along the arc will suffice and, indeed, would probably be more compact. Such a mechanism is described by Bair et al. [44] and by Namioka [43]. These scanning mechanisms could be used in the monochromator described above.

**Johnson-Onaka Mounting.** This mounting is very nearly identical to the normal incidence mountings described above. Scanning the spectrum past the exit slit is accomplished by rotating the grating about an off-axis pivot point. The point is chosen such that the grating is moved approximately along the bisector of the angle subtended by the entrance and exit slits at the grating. In the limit of small rotations the grating essentially moves along the bisector; hence the mounting is identical to those described above except that the mechanism for moving the grating is different. For larger rotations, of course, the grating center deviates more from the bisector.

Johnson [45] first described the use of an off-axis pivot point for the rotation of the grating when used near normal incidence. Onaka [46]...

†McPherson Instrument Company, Acton, Mass.
analysed the mounting in detail and showed that it was applicable for any angle of incidence with the reservation that the spectral range of good focus decreases when the angle subtended by the slits at the grating \((= 2\phi)\) deviates from the most optimum values. These values of \(\phi\) are approximately zero and 35°. It should be emphasized, however, that good focusing is achieved for any \(\phi\) over several hundred angstroms.

To locate the pivot point Onaka proceeded as follows. In Fig. 3.17 the Rowland circle focusing conditions are shown when the average wavelength \(\lambda_o\) is emerging from the exit slit \(X_o\). The entrance slit is at \(N_o\), and the grating at \(G_o\). \(M_1\) and \(M_2\) are midway points on the Rowland circle between \(N_o\) and \(X_o\). The pivot point \(C\) is chosen to lie at the appropriate position on the line joining \(G_o\) and \(M_1\). The reason for the selection of this line is that it makes a right angle with the bisector \(G_oM_2\), and for small rotations, the grating is moved towards \(M_2\) approximately along \(G_oM_2\). This movement reduces the amount of changes in the incident and emerging directions of the radiation. Thus the solution for the location of \(C\) is not a general one.

With the above constraint on the position of \(C\), its location at a distance \(l\) from \(G_o\) is found as follows. Using Beutler's equation for focusing and Rayleigh's criterion that the total pathlength for the various rays must not vary by more than \(\lambda/4\), we get

\[
|A| W^2 + B W^3 + \text{higher order approximations} \leq \frac{\lambda}{4}, \quad (3.25)
\]

where

\[
A = \frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} + \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R},
\]

and

\[
B = \frac{\sin \alpha \left( \frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} \right)}{r} + \frac{\sin \beta \left( \frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} \right)}{r'}.
\]

\(W\) is the half width of the grating, while the other terms are as previously defined. By expanding \(\alpha, \beta, 1/r, \text{ and } 1/r'\) in a power series in \(\delta\) about the initial position \(\alpha_o\) and \(\beta_o\), we get for \(A\) and \(B\),

\[
A = A_1 \delta + A_2 \delta^2 + \ldots
\]

and

\[
B = B_1 \delta + \ldots
\]

where

\[
A_1 = \frac{1}{R^2} (2l \cos \phi - R \sin \alpha_o - R \sin \beta_o - l \tan \alpha_o \sin \phi + l \tan \beta_o \sin \phi),
\]

and for small \(l\), as is the case in the extreme ultraviolet,

\[
A_2 = \frac{2 - 3 \cos^2 \phi}{R \cos \phi}, \quad (3.26)
\]

and

\[
B_1 = \frac{-2 \sin^2 \phi}{R^2 \cos \phi}. \quad (3.28)
\]

The ideal arrangement would be realized when all the coefficients \(A_1, A_2, \ldots, B_1, \ldots\) were identically equal to zero. Unfortunately, this is not possible. Thus we set the largest coefficient, namely \(A_1\), equal to zero. This gives the location of \(C\) as

\[
l = \frac{R \sin \theta}{1 - \frac{1}{2} \tan \phi (\tan \alpha_o - \tan \beta_o)}, \quad (3.29)
\]

where \(\theta = \frac{1}{2}(\alpha_o + \beta_o)\) and \(\phi = \frac{1}{2}(\alpha_o - \beta_o)\).

The wavelength range for focus within the Rayleigh criterion is given by (3.25), which now becomes

\[
W^2 \left| A_2 \delta^2 \pm B_1 W \delta \right| \leq \frac{\lambda}{4}. \quad (3.30)
\]

The \(\pm\) sign is used since the center of the grating was taken as the origin in the derivation of (3.25). Therefore \(W\), the half width of the grating, can
be either positive or negative. For a simple rotation of the grating about its center,
\[ \lambda = 2d \cos \varphi \sin (\theta + \delta) \]  
(3.31)
and for small \( \delta \),
\[ \lambda \approx 2d \cos \varphi (\sin \theta + \delta \cos \theta). \]  
(3.32)

Equation 3.32 is valid for the off-axis rotation when \( \delta \) is small. Inserting (3.27), (3.28), and (3.32) into (3.30), a quadratic equation in \( \delta \) is obtained,
\[ \xi = \frac{\omega W}{4r}. \]  
(3.35)

A monochromator with a 1 m 600 line per mm grating focused for 1000 Å and with slits subtending an angle of 9° with respect to the grating should give a useful spectral range from 370 to 1630 Å for a slit width of 40 \( \mu \)m and a grating width of 4 cm.

**Seya-Namioka Mounting.** By far the simplest scanning mechanism of any monochromator is the rotation of the grating about a vertical axis through the center of the grating. This method has been used in several cases for near normal incidence and for a limited wavelength scan [47,48]. It has proved to be satisfactory, provided a resolution of a few angstroms was acceptable. In 1952 Seya [49] analyzed the focusing conditions for a simple rotation of the grating and found that rather good focus could be expected over a large spectral range if the slits subtended an angle of approximately 70° at the grating. Namioka [50,51] constructed a monochromator based on Seya’s calculations and later analyzed the mounting in more detail with emphasis on its astigmatism and resolving power. Figure 3.19 illustrates a commercial Seya-Namioka monochromator.

The mounting is analyzed as follows. Starting with (2.9), the focal equation for the concave grating,
\[ \frac{\cos^2 \alpha R}{r} - \frac{\cos \alpha}{R} + \frac{\cos^2 \beta R}{r'} - \frac{\cos \beta}{R} = 0, \]
the constraint \( \alpha - \beta = C \) is applied, where \( C \) is a constant. This represents the constant angle subtended by the slits at the center of the grating. \( \alpha \) and \( \beta \) are of opposite sign when they lie on opposite sides of the grating normal. If we denote (2.9) by \( f \) and substitute \( \alpha - C \) for \( \beta \), then
\[ f = \frac{\cos^2 \alpha R}{r} + \frac{\cos^2 (\alpha - C) R}{r'} - \frac{\cos \alpha}{R} - \frac{\cos (\alpha - C)}{R} = 0. \]  
(3.36)

It is more convenient to express \( f \) in terms of \( \rho = R/r \) and \( \rho' = R/r' \). Thus
\[ f = \rho \cos^2 \alpha + \rho' \cos^2 (\alpha - C) - [\cos \alpha + \cos (\alpha - C)]. \]  
(3.37)
In this mounting \( \rho \) and \( C \) are constant. To maintain good focus as the grating is rotated, \( \rho' \) should also stay constant or at least change very slowly. The problem is now to select the best value of \( C \) and \( \rho \) that will make \( f \) as close to zero as possible and for which the change in \( \rho' \) is a minimum. The conditions for \( f \) to be approximately zero under the above constraints can be determined as follows.

To satisfy these three simultaneous equations, the following determinant must be zero:

\[
\begin{vmatrix}
\cos^2 \alpha_0 & \cos^2 (\alpha_0 - C) & -[\cos \alpha_0 + \cos (\alpha_0 - C)] \\
-\sin 2\alpha_0 & -\sin 2(\alpha_0 - C) & [\sin \alpha_0 + \sin (\alpha_0 - C)] \\
-2 \cos 2\alpha_0 & -2 \cos 2(\alpha_0 - C) & [\cos \alpha_0 + \cos (\alpha_0 - C)]
\end{vmatrix} = 0.
\]

(3.42)

The solutions for (3.42) are

\[
C_1 = \alpha_0 + \tan^{-1}\left(\sqrt{3} \sec \alpha_0 - 2 \tan \alpha_0\right); \quad (3.43)
\]

\[
C_2 = \alpha_0 - \tan^{-1}\left(\sqrt{3} \sec \alpha_0 + 2 \tan \alpha_0\right); \quad (3.44)
\]

\[
C_3 = 0. \quad (3.45)
\]

For any given value of \( \alpha_0 \), (3.43) and (3.45) give the values of \( C \) which satisfy the focal condition (3.36) within the approximation used in (3.38). The solution \( C_2 \) is omitted, however, since \( C_2 \) does not provide either vacuum ultraviolet or visible spectra.

To select the best value of \( C \) and \( \alpha_0 \), it is necessary that the variation in \( \rho' \) with respect to \( \alpha_0 \) be zero or at least a minimum. Consider \( C_3 \) first. Obviously \( \frac{dC_3}{d\alpha_0} \) is zero, and the optimum value of \( C_3 \) therefore remains constant during the rotation of the grating. Inserting the value \( C_3 = 0 \) into (3.39) and taking the partial derivative of \( \rho' \) with respect to \( \alpha_0 \) and equating to zero, we get

\[
\frac{\partial \rho'}{\partial \alpha_0} \bigg|_{C=\text{const}} = 2 \sec \alpha_0 \tan \alpha_0 = 0.
\]

(3.46)

This is possible only for \( \alpha_0 = 0 \). Thus the solution \( C_3 \) provides \( C = 0 \). Therefore \( \alpha = \beta \), and \( \alpha_0 = 0 \). By inspection we see that as the Rowland circle is rotated about the grating center, the exit and entrance slits both lie inside or both outside of the circle depending on the direction of rotation. This is not an optimum focusing condition as can be seen from the discussion on p. 62 above. Furthermore, the change in \( \rho' \) is excessive compared with that obtained using the solution \( C_1 \), as is discussed below. Thus the solution \( C_3 \) also will be omitted.

Considering \( C_1 \), the condition for \( \frac{dC_1}{d\alpha_0} = 0 \) is that \( \alpha_0 = 35^\circ 15' \). Inserting this value into (3.43), the value of \( C \) is found to be \( C = 70^\circ 32' \). Thus \( C \) is essentially equal to \( 2\alpha_0 \).

From (3.39) and (3.40), we find

\[
\rho' = \frac{1}{2} \sin 2\alpha_0 \sec (\alpha_0 - C) \csc C + \frac{1}{2} \sin \alpha_0 \csc C + \frac{1}{2} \sec (\alpha_0 - C).
\]

(3.47)

Substituting \( C = 2\alpha_0 \) into (3.47), \( \rho' = 1/\cos \alpha_0 \). Similarly, \( \rho = 1/\cos \alpha_0 \). That is, using the optimum value of \( C \), the initial position is such that the
optical elements all lie on the Rowland circle and \( \alpha = -\beta \). Therefore, the central image is in perfect focus at the exit slit.

To find the variation in \( \rho' \), and thus \( r' \), under the above conditions for \( C_1 \) consider the spectral range from 0 to 2000 \( \text{Å} \) with \( R = 1 \text{ m} \), \( 1/d = 600 \) lines/mm, \( m = +1 \), \( \rho = 1/\cos \alpha_0 \), \( \alpha_0 = 35^\circ 15' \), and \( C = 70^\circ 30' \). To cover this range the grating must be rotated through an angle \( \Delta x = 4^\circ 13' \) (0.07 radians). Substituting these values into (3.36), the variation in \( r' \) is \( \Delta r' \sim 0.007 \text{ mm} \). For other spectral ranges the best values of \( C, \rho, \) and \( \rho' \) have been calculated by Greiner and Schäffer [52]. Using the same parameters as above, we find that the solution \( C_3 = 0 \) gives \( \Delta r' \sim 1.8 \text{ mm} \), hence the reason for omitting this solution.

In the Seya-Namioka mounting the constructional parameters are

\[
C = 70^\circ 30' \ ;
\rho = \rho' = \frac{1}{\cos 35^\circ 15'} = 1.2245.
\]

Because these parameters are independent of \( R \) and \( d \), any grating can be used in a given monochromator by simply changing the lengths of the entrance and exit arms such that \( \rho \) and \( \rho' \) stay constant, a higher resolution being achieved for increasing \( R \) and decreasing \( d \). Figure 3.20 shows the variation in widths of the spectral lines at half their maximum intensities as a function of wavelength for \( R = \frac{1}{2} \text{ m} \) and \( 1/d = 1200 \text{ lines per mm} \).

The small amount of defocusing caused by the variation in \( r' \sim 0.007 \text{ mm} \) and for small deviations in the parameters \( \rho, \rho' \), and \( C \) is negligible compared to the higher order aberrations in the grating equation, which are zero in the Rowland and Wadsworth mounting of the grating but not zero in the Seya-Namioka mounting. In fact, the astigmatic image is slightly curved and is the major cause for poor resolution when long, straight exit slits are used. The use of slightly curved exit slits greatly improves the resolution. The curvature of the spectral image is caused primarily by the aberration called **astigmatic curvature** discussed in Chapter 2. The radius of curvature of the exit slit should be made equal to \( R_0/\sin \) (see (2.28)).

For the Seya mounting the curvature of the spectral lines remains nearly constant between 0 and 2000 \( \text{Å} \). Thus the radius of the exit slit is given by

\[
r = 0.577R.
\]

The center of curvature lies between the entrance and exit slits. Bath and Brehm [53] have obtained a wavelength resolution of 0.2 \( \text{Å} \) using a 1 m grating with 1440 lines per mm and curved slits 2 \text{ cm} long and 20° wide. The author has obtained a resolution of 0.6 \( \text{Å} \) using a 0.5 m grating with 1200 lines per mm with a curved exit slit \( (r = 28.5 \text{ cm}) \) 1.3 cm high by 25° wide. The astigmatism in the Seya mounting is approximately

\[
z = l + \frac{1}{2}L,
\]

where \( l \) is the height of the entrance slit and \( L \) is the length of the grating rulings illuminated. Equation 3.49 is found to hold in practice. This astigmatism can be greatly reduced by using a toroidal grating with radii of ratio \( R/R_n \sim 0.67 \) (see Sect. 2.6).

The theoretical resolving power of the monochromator is a maximum for a grating width of approximately 2.8 cm using a 1 m grating with 600 lines per mm. For wider gratings, the resolution rapidly decreases. The intensity, on the other hand, increases with grating width until a width of approximately 4 cm is reached, at which point the intensity approaches a constant value. Thus the width of the grating must be chosen to optimize the resolution or the intensity.

The best scanning mechanism for a Seya monochromator is the sine drive, which produces a linear wavelength scale as the grating is rotated. For a simple rotation of the grating through an angle \( \theta \), the grating equation can be expressed as \( m\lambda = 2d \cos \varphi \sin \theta \), where \( \varphi \) is equal to \( \frac{1}{2}(\alpha - \beta) \) and \( \theta \) is equal to \( \frac{1}{2}(\alpha + \beta) \). When the exit and entrance slits are fixed, \( \varphi \) is constant. Therefore the wavelength appearing at the exit slit is
proportional to the sine of the angle of rotation. A drive which provides a linear displacement proportional to \( \sin \theta \) is called a sine drive [54,55].

Miyake and Katayama [56–58] have discussed the theory for off-Rowland circle mountings more generally and have derived all the preceding mounts as special cases.

**Plane Grating Mountings**

**Czerny-Turner and Ebert-Fastie Mounts.** The Ebert-Fastie mounting [19–21] has been described under spectrographs in Sect. 3.1. However, its major advantage as described by Fastie is in its use as a scanning spectrometer. This instrument is basically a special case of the more general mounting described by Czerny and Turner [22]. Figure 3.21 shows the optical system of the single mirror as used in the Ebert-Fastie monochromator while Fig. 3.22 shows the two-mirror system used by Czerny and Turner. The monochromatic action is achieved by a simple rotation of the plane grating about its center.

In both systems the off-axis aberrations produced by the reflection from the first concave mirror are cancelled by the reflection from the second concave mirror. In fact, it appears that the limit of resolution is determined by imperfections in the grating and not by aberrations of the optical system. To obtain maximum resolution from these systems Fastie has shown that curved exit and entrance slits should be used, the radius of curvature being equal to the separation distance of the slits from the optical axis of the system. This circle is called the Ebert circle. Thus, the ability to use very long slits without impairing the resolution makes this mounting particularly suitable for observing extended light sources.

The three reflections required in a plane grating mounting are extremely troublesome in the vacuum ultraviolet region since the reflecting power of most materials decreases rapidly below 2000 A. However, the combination of aluminum and magnesium fluoride coatings has proved to be an excellent reflector for radiation down to about 1200 A [59,60]. With such high reflecting coatings, the Czerny-Turner and Fastie-Ebert monochromators probably have the most desirable features when working above 1200 Å.

**Collimator Mounting.** With collimators, instead of the conventional exit and entrance slits, and with a plane diffraction grating, it is possible to construct a very simple monochromator of moderate resolution. Monochromators of this type have often been used in the x-ray region. Until recently, however, none had been used in the vacuum uv region [62]. Figure 3.23 shows the arrangement of the components for the type of collimating monochromator constructed by Bedo and Hinteregger [62]. The wavelengths can be scanned by a simple rotation of the plane grating about an axis in the surface of the grating parallel to the rulings. The collimator is constructed using a series of grids spaced appropriately to achieve maximum collimation. Collimation can also be applied to the incident beam if it is not sufficiently parallel. Each grid consists of a series of slits etched by an electroforming process. The arrangement of
The zone plate is an array of concentric circles whose radii are approximately proportional to the square roots of the consecutive integers 1, 2, 3, ..., while the area between consecutive circles are made alternatively opaque and transparent to the incident radiation. The principle of the zone plate is described in most optical textbooks [63], while the theory has been discussed recently by Myers [64] and Kamiya [65].

Briefly, the principle of the zone plate is shown in Fig. 3.25 for parallel light. The radii of the circles are given by \( r_1, r_2, \ldots \), etc., and are constructed so that the distance from the circumference of a given circle to the point \( P \) differs from that of a consecutive circle by \( \lambda/2 \). Therefore, if every alternate zone is made opaque to the incident radiation, then the radiation penetrating the open apertures and directed towards \( P \) will on the average have pathlengths differing by \( n\lambda \), \( n = 1, 2, 3, \ldots \), and the light intensity will be reinforced at \( P \). It makes no difference whether the central zone is opaque or transparent: the focusing properties of the zone plate are the same. The optical properties of the zone plate are as follows:

- **Radius** of 1\(^{st}\) ring:
  \[ r_1^2 = f n \lambda + \frac{n^2 \lambda^2}{4}, \quad n = 1, 2, 3, \ldots \]  
  (3.50)

where \( f \) is the focal length of the zone plate for a wavelength \( \lambda \);

- **Focal Length**:
  \[ f = \frac{r_1^2}{m \lambda}, \quad m = 1, 3, 5, \ldots \]  
  (3.51)

when \( \lambda/4 \ll f \), and where \( m \) is the order number. The primary image is given when \( m = 1 \) and is the most intense image. The images appearing
for \( m = 3, 5, \ldots \) are less intense and are approximately in the ratio \( 1:m \). A second order image \( m = 2 \) may be observed but is due to any inaccuracy in the construction of the zone plate.

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \quad (3.52)
\]

where \( p \) and \( q \) are, respectively, the object and image distances.

**Minimum Angle of Resolution**

\[
\theta = \frac{1.22\lambda}{D}, \quad (3.53)
\]

where \( D \) is the diameter of the zone plate.

**Dispersion**

\[
\frac{dq}{d\lambda} = \frac{mr_1^2}{(m\lambda - r_1^2/p)^2} \quad (m = 1, 3, 5, \ldots) \quad (3.54)
\]

\[
= \frac{r_1^2}{m\lambda^2}, \quad \text{for} \quad p \to \infty. \quad (3.55)
\]

For the zone plate to be of practical use in the vacuum ultraviolet, the transparent zones must be completely open. Baez [66] has succeeded in constructing such a zone plate with 38 zones. The opaque elements were thin concentric bands of gold made self-supporting by the use of thin radial struts, as shown in Fig. 3.26. The central circle had a diameter of 0.0426 cm, while the outer circle and diameter of the zone plate was 0.2596 cm. The error in the circle diameters was \( \pm 0.0002 \) cm.

Baez and Myers have suggested the use of zone plates as a focusing device for extreme uv and x-rays. Equation 3.52 above shows that the zone plate has focal properties which are similar to a simple lens. Using the open construction type zone plate, Baez has photographed the image of a mesh with 4 lines per mm illuminated with light of wavelength 6700 Å, 4538 Å, and 2537 Å. Figure 3.27 shows the images normalized to the same magnification. The resolution of the images clearly increases as the wavelength decreases. This was predicted by (3.53). It would be expected, therefore, that excellent resolution would be obtained at shorter wavelengths. Baez has shown that the zone plate, compared to a pinhole camera, has higher resolution and greater light-gathering power. In fact, the optimum diameter of a pinhole is given by [67]

\[
D = 2\sqrt[4]{\frac{0.9pq\lambda}{p + q}}, \quad (3.56)
\]

![Fig. 3.26 Photograph of a self-supported gold zone plate. The diameter of the outer circle is 0.26 cm while the diameter of the central circle is 0.043 cm. The thickness of gold is estimated as 10 microns. The white bands represent the completely open structure (courtesy A. V. Baez [66]).](image1)

![Fig. 3.27 Zone plate images of a mesh with 4 lines per mm illuminated with radiation of wavelength 6700, 4538, and 2537 Å (courtesy A. V. Baez [66]).](image2)
slits will lie on the circumference of the Rowland circle. If the center of the Rowland circle has been located previously, the positioning of the grating normal is straightforward since it must pass through the center of the Rowland circle. The procedure used by McPherson is to stretch a thin wire vertically over the grating mount, passing over its center, and then to locate a conical bob at the center of the Rowland circle. The point of the bob is lined up visually with the vertical wire, while the grating is rotated about its vertical axis until the image of the bob is also in line with the wire, bob, and eye.

The plateholder is machined to follow the curvature of the Rowland circle to as high a degree as possible. It can be helpful if the plateholder is pivoted about the point of intersection of the grating normal with the Rowland circle since the plateholder can then be rotated about this point until it coincides with the Rowland circle. This point can be located rather precisely by using the Foucault knife-edge test. The principle of this test is that if the image of a small illuminated hole is observed at the focal plane, then the grating is seen to be filled with light; and if a knife-edge is passed through the focal plane, the entire grating should suddenly and uniformly become dark. When the knife edge is moved across the wedge of light a short distance inside the focus, its shadow, as it appears on the grating, moves in the same direction as the knife-edge; if it is placed outside the focus, the shadow moves in the opposite direction. The knife-edge test is remarkably sensitive and can be used to locate the position of the focal plane to within approximately 0.1 mm.

Where tests must be made on the nonvisible lines, the "hour-glass" technique can be used. This method is illustrated in Fig. 3.28. A wedge is made with the sloping face inclined at a few degrees to the horizontal.
Then by mounting a photographic film onto the inclined face, the wedge is inserted into the diffracted beam. An image shaped like an hour-glass results. The narrow neck of the image is the position of the focal plane. A datum line can be engraved on the wedge indicating the plane of the plateholder. The deviation between the datum line and the narrowest part of the image indicates the amount and direction of the correction. Using this technique, Shenstone [68] has observed in his spectrograph that while the first, second, and fourth order spectra were sharply focused, the third order was not. This anomaly is not well understood but may be common to most gratings. Thus when optimum focusing is required it is advisable to make final adjustments with the wavelengths and spectral order of interest.

In general, it is desirable to use precision machining to locate the position of the optical elements on the Rowland circle. This is especially true in the grazing incidence mounting where the construction is such that the grating normal is not always accessible. When this is done, only minor adjustments are necessary to bring the elements into their best position for good focus.

A detailed discussion of the adjustments necessary in grating spectrographs is given by Harrision, Lord, and Looibourow [69], while Sawyer [70] and MacAdam [71] discuss the optics of small displacements from the Rowland circle.

REFERENCES

[34] Y. Fujikawa and R. Ito, Sci. of Light (Tokyo) 1, 1 (1951).
[35] A 40-cm radius mounted monochromator is commercially available from the Tropel Co.
Vacuum Techniques

The degree of vacuum necessary in a spectrograph to prevent any appreciable attenuation of the radiation from the light source depends on the wavelength and the pathlength from the entrance slit to the plateholder via the diffraction grating. The amount of attenuation of the radiation is given by the Lambert-Beer Law, namely,

$$ I = I_0 e^{-\alpha n L}, $$

(4.1)

where $I_0$ is the intensity of the unattenuated beam while $I$ is the intensity reaching the plateholder or exit slit; the pathlength traveled by the radiation is $L$; the number of molecules in the spectrograph is represented by $n$ per cubic centimeter, and the absorption cross section of the molecules (air) is represented by $\alpha$. The absorption cross section of a gas is a function of wavelength and in general is greater at wavelengths shorter than the first ionization potential of the gas. Since the first ionization potential of most gases lie below 1100 Å, and no window materials exist below 1040 Å, the minimum vacuum requirements are divided naturally into two regions above and below 1100 Å. The region below 1100 Å not only requires a better ultimate vacuum than the region above 1100 Å, but it also requires higher pumping speeds to remove the gas, which continually diffuses in through the entrance slit from the light source. Pressures of $10^{-4}$ to $10^{-6}$ torr are quite adequate for most vacuum spectrographs and monochromators.

The choice of vacuum pumps, both mechanical and vapor types, is determined mainly by the following: the desired pump down time from atmospheric pressure to an operational pressure; the desired ultimate vacuum; and the pumping speed necessary to handle gas leakage from the light source, windowless absorption cells, and any spurious leaks in the system.

The time taken to exhaust a volume $V$, with no leaks and with no appreciable outgassing, from a pressure $P_1$ to $P_2$ is given by

$$ t = 2.3 \frac{V}{S} \log \frac{P_1}{P_2}, $$

(4.2)