MAGNETO-OPTICS AND ELECTRO-OPTICS

We have already seen in Chap. 20 and Secs. 23.9, 26.9, and 28.9 that the electromagnetic theory is capable of explaining the main features of the propagation of light through free space and through matter. In further support of the electromagnetic character of light, there is a group of optical experiments which demonstrates the interaction between light and matter when the latter is subjected to a strong external magnetic or electric field. In this group of experiments those which depend for their action on an applied magnetic field are classed under magneto-optics and those which depend for their action on an electric field are classed under electro-optics. In this the following known optical effects will be treated briefly under these headings:

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32.1 ZEEMAN* EFFECT

In the year 1896, Zeeman discovered that when a sodium flame is placed between the poles of a powerful electromagnet, the two yellow lines are considerably broadened. Shortly afterward, Lorentz presented a simple theory for these observations, based upon the electron theory of matter, and predicted that each spectrum line when produced in such a field should be split into two components when viewed parallel to the field [Fig. 32A(a)] and into three components when viewed perpendicular to the field [Fig. 32A(b)]. He further predicted that in the longitudinal direction (a) these lines should be circularly polarized and in the transverse direction (b), plane-polarized. With improved experimental conditions these predictions were later verified by Zeeman, Preston, and others in the case of some spectral lines.

The Lorentz theory assumes that the electrons in matter are responsible for the origin of light waves and that they are charged particles whose motions are modified by an external magnetic field. In the special case of an electron moving in a circular orbit, the plane of which is normal to the field direction B, the electron should be speeded up or slowed down by an amount proportional to the magnetic induction B. A classical treatment of this problem shows that if $v_o$ represents the orbital frequency of the electron in a field-free space, the frequency in the presence of a field will be

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* P. Zeeman (1865-1943). Dutch physicist and Nobel prize winner (1902). He is most famous for his work on the splitting up of spectral lines in a magnetic field. His chief contributions are summarized in his celebrated book "Researches in Magneto-optics," Macmillan Co., Ltd., London, 1913.
given by $v_0 \pm \Delta v$, where

\begin{equation}
\Delta v = \frac{eB}{4\pi m} = 1.399611 \times 10^{10} B \ s^{-1} \tag{32a}
\end{equation}

where $e$ is the charge on the electron in coulombs, $m$ is the electron mass in kilograms, and $B$ is the magnetic induction in teslas. One tesla = 1 T = 1 Wb/m² = 10,000 G.

In the study of spectrum lines this frequency difference $\Delta v$ is most conveniently expressed in wave numbers (see Sec. 14.14) by dividing by the speed of light in centimeters per second; $c = 2.997925 \times 10^{10}$ cm/s:

\begin{equation}
\Delta \sigma = \frac{\Delta v}{c} = 0.46686 \ B \ cm^{-1} \tag{32b}
\end{equation}

A useful relationship between wavelength and frequency in hertz or wave numbers follows from the wave equation $c = \nu \lambda$:

\begin{equation}
\frac{\Delta \lambda}{\lambda} = \frac{\Delta \nu}{\nu} = \frac{\Delta \sigma}{\sigma} \tag{32c}
\end{equation}

where $\Delta \lambda$ is small compared with $\lambda$, $\Delta \nu$ is small compared with $\nu$, and $\Delta \sigma$ is small compared with $\sigma$.

In the classical theory of the Zeeman effect we are concerned with an aggregation of atoms in which the electrons are revolving in circular or elliptical orbits oriented at random in space. It will now be shown, however, that this situation is equivalent to having one-third of the electrons vibrating in straight lines along the direction of the magnetic field and two-thirds of them revolving in circular orbits in the plane perpendicular to the field. Of the latter ones, half are revolving in one sense and half in the opposite sense. The radius of their orbits is $1/\sqrt{2}$ times the amplitude of the linear vibrations. To prove these statements, let us select any one of the electrons and resolve its elliptical motion into three mutually perpendicular linear motions as shown in Fig. 32B(a). For simplicity we shall assume that the electron is bound by an elastic force obeying the law

\begin{equation}
F = -kr \tag{32c}
\end{equation}

where $r$ is the displacement from the equilibrium position. Under this condition the three components are simple harmonic motions, but for any one electron they are not equal in amplitude or in the same phase.

If a magnetic field is now applied in the $z$ direction, the component parallel to $z$ will be uninfluenced, for it is equivalent to a current directed along the lines of force. The $x$ and $y$ vibrations will each be modified, however, since an electron which is moving across a magnetic field experiences a force

\begin{equation}
F_B = Bev \tag{32d}
\end{equation}

perpendicular to the field and also perpendicular to its motion. The effect of this force is to change the $x$ and $y$ components into rosette motions such as that shown in Fig. 32B(b) for the $y$ component. These can be described to better advantage in terms of circular components, $y^+$ and $y^-$ for the $y$ motion, and $x^+$ and $x^-$ for the $x$ motion [diagram (c) of the figure]. In the presence of the field both plus circular components have a higher frequency than the minus ones, so we may combine the $x^+$ and $y^+$ motions to get a resultant positive circular motion, as in diagram (d),
Thus the original elliptical orbit when subjected to a magnetic field is equivalent to a linear motion of unchanged frequency along the field, plus two circular motions, one of higher and one of lower frequency, in the plane at right angles to the field.

Only the circular components will emit light along the field direction, and these give circularly polarized light of two different frequencies. The intensity of these two components must be equal when the whole aggregation of atoms is considered, because as the field goes to zero, the light is unpolarized. When we observe the light at right angles to the field, we are viewing the circular components edge-on, so these yield two different frequencies of plane-polarized light in which the vibrations are perpendicular to the field direction. Each of them has only half the intensity of the above-mentioned circularly polarized beams. In addition, the linear z motions emit light in the transverse direction. This light has the original frequency $v_0$, vibrates parallel to the field, and has an intensity equal to the sum of the other two. The mean amplitude of the z components for all atoms is therefore $\sqrt{2}$ times as great as that of the x or y components.

Now let us calculate the change of frequency to be expected for the circular components. In the absence of the field the centripetal force on the electron in its circular orbit is furnished by the elastic force, so that by Eq. (32c) we have

$$F = - kr = - m\omega_0^2 r \quad (32e)$$
being the mass of the electron and \( \omega \), its angular velocity. After the field is applied, there is a new angular velocity \( \omega' \), and the new centripetal force must be the sum of the elastic force and the force due to the field (Eq. 31d). Thus

\[
F' = -m\omega'^2r = F + F_B = -kr \pm Bev
\]

The positive sign corresponds to a clockwise rotation in the \( xy \) plane and the negative sign to a counterclockwise one. Substituting for \(-kr\) its value from Eq. (32e), we then obtain

\[
-m\omega r = -m\omega_0^2r \pm Bev
\]

or, since \( v/r = \omega \),

\[
\omega^2 - \omega_0^2 = \pm \frac{Benv}{mr} = \pm \frac{Benv}{m} \quad (32f)
\]

In order to get a simple expression for the change of frequency, it is necessary to assume that the difference in the \( \omega \)'s is small compared to either \( \omega \). This is always justified in practice since it means that the Zeeman shifts are small compared with the frequency of the lines themselves. Then we may put

\[
(\omega + \omega_0)(\omega - \omega_0) \approx 2\omega(\omega - \omega_0)
\]

and, from Eq. (32f),

\[
\omega - \omega_0 = \pm \frac{Be\omega}{m^2\omega} = \pm \frac{Be}{2m}
\]

Since \( \nu = \omega/2\pi \), the change in frequency becomes

\[
\Delta \nu = \pm \frac{Be}{4\pi m} \quad (32g)
\]

in agreement with Eq. (32a).

In this derivation it has been tacitly assumed that the radius of the circular motion remains unchanged during the application of the magnetic field. The speeding up or slowing down of the electron in its orbit occurs only while the field is changing and is due to the changing number of lines of force threading the orbit. By Faraday's law of induction this change produces an emf just as it would in a circular loop of wire. The resulting increase or decrease of velocity might be expected to change the radius, but the fact is that there is a corresponding alteration in the centripetal force which is just sufficient to maintain the radius constant. The additional force is that represented by Eq. (32d), which has the same origin as the perpendicular force on a wire carrying a current in a magnetic field.

Let us now summarize what should be the observed effect of a magnetic field on a spectrum line. The result will depend on the direction, with respect to that of the magnetic field, in which the source is viewed. When the source is viewed in the direction of the field, along the \( z \) axis, we have what is called the longitudinal Zeeman effect. From this direction only the frequencies \( \nu_0 + \Delta \nu \) and \( \nu_0 - \Delta \nu \) should appear,
Zeeman patterns for a normal triplet, showing the polarization of the light.

and this light should be right- or left-handed circularly polarized* [Fig. 32C(a)]. Since light is a transverse wave motion, the $z$ vibrations will not emit light of frequency $\nu_0$ in the $z$ direction. Viewed perpendicular to the field, the $z$ motions should be observed to give plane-polarized light with the electric vector parallel to the field ($p$ components), and the circular motions, seen edge-on, should give plane-polarized light with the electric vector perpendicular to the field ($s$ components). A spectrum line viewed normal to $B$ should therefore reveal three plane-polarized components [Fig. 32C(b)]—a center unshifted line, and two other lines symmetrically located as shown. This is called a normal triplet and is observed for some spectrum lines, though by no means the majority of them.

Since the direction of rotation of the circularly polarized light depends on whether one assumes positive or negative charges as the emitters of light, it is possible to distinguish between these alternatives by using a quarter-wave plate and nicol. Figure 32C(a), where the positive rotation has the higher frequency, was drawn according to our assumption of negative electrons as the emitters.

In Zeeman's early investigations he was not able to split any spectrum lines into doublets or triplets, but he did observe that they were broadened and that the outside edges were polarized, as predicted by Lorentz. The polarization corresponded to emission by negative particles. He was later able to photograph the two outer components of lines arising from the elements zinc, copper, cadmium, and tin, by cutting out the $p$ components with a nicol prism. Preston, using greater dispersion and resolving power, was able to show not only that certain lines were split up into triplets when viewed perpendicularly to the field but that others were split into as many as four, five, or even a much larger number of components. Such patterns of lines, shown in Fig. 32D, are called anomalous Zeeman patterns, and the phenomenon is called the anomalous Zeeman effect. The normal triplet separation $2\Delta\nu$ as

* Using the right-hand rule with the thumb pointing in the direction of the field, the fingers point in the direction of the + rotations which have the higher frequency designated by $\nu_1$. The opposite direction gives the − rotations with the lower frequency $\nu_2$. Looking against the light, clockwise rotations give rise to right-handed polarized light and counterclockwise rotations give rise to left-handed circularly polarized light. This latter is in agreement with the definitions used in treating optically active substances.
given by the classical theory is shown by the bracket below each pattern. From Eq. (32a) it is seen that each of the outer component lines should shift out by an amount proportional to the field strength, thus keeping the pattern symmetrical. In very strong magnetic fields, however, asymmetries are observed in many Zeeman patterns. This phenomenon is known as the quadratic Zeeman effect, although it may also be the beginning of a transition called the Paschen-Back effect according to which all anomalous patterns become normal triplets in the limit of very strong fields.

Only the normal triplet can be explained by the classical theory. The more complex patterns are now understood and are in complete agreement with the quantum theory of atomic structure and radiation.* Each line of an anomalous pattern, when viewed perpendicular to the magnetic field, is found to be plane-polarized. Usually the centerlines of a pattern are \( p \) components with their vibrations parallel to the field \( B \), and those symmetrically placed on either side are \( s \) components with vibrations perpendicular to the field. In the longitudinal effect only frequencies corresponding to the \( s \) components are observed, and these are circularly polarized.

The quantum theory has developed to such an extent that one can now predict

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with the greatest certainty the complete Zeeman pattern for any identified spectrum line in a field of any strength. Conversely, the study of these patterns has become a potent tool in the analysis of complex spectra.

### 32.2 INVERSE ZEEMAN EFFECT

The Zeeman effect obtained in absorption is called the inverse Zeeman effect. The phenomenon is observed by sending white light through an absorbing vapor when the latter is subjected to a uniform magnetic field. In considering the longitudinal effect, analogous to Fig. 32C(a), unpolarized light of any particular frequency may be imagined as consisting of right and left circularly polarized components with all possible phase relations. If now $v_0$ represents a natural resonance frequency of the vapor in the absence of a field, the plus circular components (see footnote, page 683) of frequency $v_1$ will be strongly absorbed in the presence of a field. The corresponding minus circular components of frequency $v_1$ pass on through with little decrease in intensity, since to be absorbed these must have the frequency $v_2$. Hence at frequency $v_1$, looking against the field direction as in Fig. 32C(a), right circularly polarized light is transmitted, and for a thick absorbing layer this is one-half as intense as the background of continuous light [Fig. 32E(a)]. A similar argument can be given for $v_2$.

The Zeeman components of any spectrum line obtained in absorption along the field direction are therefore not completely absorbed, and the light that does get through is found to be circularly polarized in directions opposite to those of the corresponding components obtained in emission. This is verified by experiment even in anomalous patterns of many components.

Viewed perpendicular to the field [Fig. 32E(b)], the $p$ and $s$ components are polarized at right angles to the corresponding components in emission. For $v_0$, the parallel components of all incident light vibrations are absorbed and the perpendicular components are transmitted. For $v_1$, the parallel components are all transmitted. The perpendicular components, moving across the field, are absorbed by only half the oscillators (the ones having positive rotation, frequency $v_1$), giving an absorption line only half as intense as that at $v_0$. The result is partially polarized light with a maximum intensity for vibrations parallel to the field $B$. The same is true for the
component $v_2$. The absorption of the parallel component for $v_0$ is analogous to the selective absorption in crystals like tourmaline (Sec. 24.6), where one component vibration is completely absorbed and the other transmitted. The frequencies of the lines observed in the inverse Zeeman effect are also given by Eqs (32a) and (32b).

### 32.3 FARADAY EFFECT

In 1845 Michael Faraday discovered that when a block of glass is subjected to a strong magnetic field, it becomes optically active. When plane-polarized light is sent through glass in a direction parallel to the applied magnetic field, the plane of vibration is rotated. Since Faraday’s early discovery the phenomenon has been observed in many solids, liquids, and gases. The amount of rotation observed for any given substance is found by experiment to be proportional to the field strength $B$ and to the distance the light travels through the medium. This rotation can be expressed by the relation

$$\theta = VBl$$  \hspace{1cm} (32h)

where $B$ is the magnetic induction in teslas, $l$ is the thickness in meters, $\theta$ is the angle of rotation in minutes of arc, and $V$ a constant to be associated with each substance. This constant, called the Verdet constant, is defined as the rotation per unit path per unit field strength. In gases the density must also be specified. A few values of the Verdet constant are given in Table 32A.

The Faraday effect is so closely associated with the direct and inverse Zeeman effects, presented in the two preceding sections, that its explanation follows directly from the principles given there. Because the phenomenon is best observed in vapors at wavelengths near an absorption line, the explanation given here will be confined to substances in the gaseous state. Consider the passage of light through a vapor like sodium where in the absence of a field there are certain resonance frequencies $v_0$ at each of which absorption takes place. When the magnetic field is introduced, there will be for each $v_0$, according to the classical theory of the Zeeman effect, two resonance frequencies, one $v_1$ for left circularly polarized light and the other $v_2$ for

<table>
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<tr>
<th>Substance</th>
<th>$t$, °C</th>
<th>$V$</th>
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</thead>
<tbody>
<tr>
<td>Water</td>
<td>20</td>
<td>$1.31 \times 10^4$</td>
</tr>
<tr>
<td>Glass (phosphate crown)</td>
<td>18</td>
<td>$1.61 \times 10^4$</td>
</tr>
<tr>
<td>Glass (light flint)</td>
<td>18</td>
<td>$3.17 \times 10^4$</td>
</tr>
<tr>
<td>Carbon disulfide (CS$_2$)</td>
<td>20</td>
<td>$4.23 \times 10^4$</td>
</tr>
<tr>
<td>Phosphorus, P</td>
<td>33</td>
<td>$13.26 \times 10^4$</td>
</tr>
<tr>
<td>Quartz (perpendicular to axis)</td>
<td>20</td>
<td>$1.66 \times 10^4$</td>
</tr>
<tr>
<td>Acetone</td>
<td>15</td>
<td>$1.109 \times 10^4$</td>
</tr>
<tr>
<td>Salt (NaCl)</td>
<td>16</td>
<td>$3.585 \times 10^4$</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>25</td>
<td>$1.112 \times 10^4$</td>
</tr>
</tbody>
</table>
FIGURE 32F
Absorption and dispersion curves used in explaining the Faraday effect. These curves refer to the magnetic splitting of a single absorption line.

right circularly polarized light traveling along the field. For each of these directions of rotation an absorption curve and a dispersion curve [Fig. 23H(b)] may be drawn as shown in Fig. 32F(a) and (b).

Referring to Fig. 32F(b), it is observed that outside the region \( v_1 \) to \( v_2 \) the value of \( n^- \) is greater than \( n^+ \). Therefore positive rotations travel faster than negative, and the plane of the incident polarized light is rotated in the positive direction (see Sec. 28.3). The difference between the two dispersion curves, as given in Fig. 32F(c), shows that for frequencies between \( v_1 \) and \( v_2 \) the rotation is in the negative direction.

If plane-polarized light is reflected back and forth through the same magnetically activated vapor, the plane of vibration is found to rotate farther with each traversal. This is not the case for naturally optically active substances like quartz, where upon one reflection the light emerges vibrating in the same plane in which it entered. It should be noted that when the field direction is reversed, the direction of rotation of the plane of the incident light vibrations is also reversed. Therefore the sense of the rotation is defined in terms of the direction of the field, positive rotation being that of a right-handed screw advancing in the direction of the field, or that of the positive current in the coil which produces the field.

The rotation in the Faraday effect is given by Eq. (32h), which shows that the angle of rotation is proportional to the field strength. This follows from Eq. (32a) for the Zeeman effect. As the two dispersion curves separate with an increasing field, the differences in index (bottom curve) increase to a first approximation by an amount which is proportional to \( \Delta \nu \) and hence to \( B \). This is most accurately true at frequencies far from \( v_1 \) or \( v_2 \), where the dispersion curves over a short frequency interval may be considered as straight lines.

One of the most interesting methods developed for observing the Faraday effect is that shown in Fig. 32G. Without the right- and left-handed quartz prisms or the vapor, no light would be transmitted by the analyzer when crossed by the polarizer as shown. With the insertion of the double quartz prism the light vibrations are rotated by different amounts according to the portion of the prisms (in the plane of the figure) through which they have passed. Hence varying amounts of light will
now get through the various portions of the analyzer. When this light is focused on the slit of a spectroscope, alternate dark and light bands are formed as shown in Fig. 32G(b). If white light is used as a source in front of the polarizer, the spectrum as observed in the spectroscope will be crossed by a number of approximately horizontal dark and light bands. If now the vapor is introduced into the light path, absorption lines will be observed at all resonance frequencies $\nu_0$. When the magnetic field is turned on, rotation takes place within the vapor according to Fig. 32F(c), thus shifting the bright bands accordingly. Close to the absorption lines the rotation is large, causing greater shifts of the bands. Since this rotation changes continuously with $\lambda$, the bands are observed to curve up or down, taking the same general form as shown in the theoretical curve of Fig. 32F(c). Figure 32H(a) is a photograph of these bands for the D lines of sodium taken under high dispersion and resolving power. They show not only the rapid increase in the positive rotation on each side of the absorption frequencies but the opposite rotation between the two. It should be noted that both these sodium lines give anomalous Zeeman patterns [Fig. 32D(b)]. The longitudinal effect for $\lambda 5896$, D$_1$, however, is a doublet leading to the same kind of curves as those described above for a normal triplet. Theoretical curves for the D$_2$ line are left as an exercise for the student.

### 32.4 VOIGT EFFECT, OR MAGNETIC DOUBLE REFRACTION

In 1902 Voigt discovered that when a strong magnetic field is applied to a vapor through which light is passing perpendicular to the field, double refraction takes place*. This phenomenon, now known as the Voigt effect or magnetic double refraction, is related to the transverse Zeeman effect in precisely the same way that the Faraday effect is related to the longitudinal Zeeman effect. In view of this relation the phenomenon is readily explained from absorption and dispersion curves in much the same

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way as the Faraday effect in the preceding section. Consider a vapor having a reso-
nance frequency $v_0$ which in the presence of an external field breaks up into a normal
Zeeman triplet [see Fig. 32C(b)]. When white light is sent through this vapor, those
light vibrations which have a frequency $v_0$ will be in resonance with electrons of the
vapor which have $v_0$ as their frequency and thus be absorbed. This is represented
by the central absorption and dispersion curve in Fig. 32I(a) and (b). Other light
vibrations, perpendicular to the field, are in resonance with $v_1$ and $v_2$. These are
represented by the $\perp$ absorption and dispersion curves. With unpolarized light
incident on the vapor the variations in $n$ near $v_1$ and $v_2$ are half as great as at $v_0$,
just as the absorption coefficients at $v_1$ and $v_2$ are only half as great as at $v_0$.

The dispersion curves of Fig. 32I(b) show that if plane-polarized light of any
frequency $\nu$ is incident on the vapor it will be broken up into two components, one
perpendicular and one parallel to $B$. Since these components have different refractive
indices (therefore different velocities), one component gets ahead of the other in
phase and elliptically polarized light emerges. The relative magnitude of this phase
difference varies with wavelength, as is shown by the difference curve in Fig. 32I(c).
To observe the Voigt effect, an experiment may be set up as shown for the Faraday effect in Fig. 32G. The field should be turned perpendicular to the absorption tube and the quartz double prism replaced by a Babinet compensator (Fig. 27F). Without the absorption tube the spectroscope slit and the photographic plate will be crossed by parallel light and dark bands. When the vapor is introduced, absorption is observed at $\nu_0$. When the field is turned on, strong double refraction close to $\nu_0$, $\nu_1$, and $\nu_2$ causes these bands to curve up or down as shown in the photographs of Fig. 32H(b) and (c). This pattern in (c) is a normal triplet observed in the Zeeman effect of the lithium spectrum.*

The Voigt effect for anomalous Zeeman patterns like those in Fig. 32H(b) has been studied by Zeeman, Geest, Voigt, Landenberg, Hansen, t and others. These results are readily predicted by drawing dispersion curves similar to those shown in Fig. 321. In any Zeeman pattern the $s$ components form one continuous dispersion curve and the $p$ components another. Their difference represents a plot of the double refraction as a function of the frequency. Its magnitude is proportional to the square of the field strength $B$.

### 32.5 Cotton-Mouton Effect

This effect, which was discovered in 1907 by Cotton and Mouton, has to do with the double refraction of light in a liquid when placed in a transverse magnetic field. In pure liquids like nitrobenzene very strong double refraction is observed, the effect being some thousand times as great as the Voigt effect treated in the last section. This double refraction is attributed to a lining up of the magnetically and optically anisotropic molecules in the applied field direction. This lining up would result

* The lithium line $\lambda 6707$ is in reality a doublet, each component of which in a weak magnetic field gives an anomalous Zeeman pattern. In the strong field used to observe the Voigt effect the two have coalesced (the Paschen-Back effect) to form a normal triplet for which the above discussion has been given.

whether the magnetic dipole moments of the molecules were permanent or induced by the field. Such an effect should be by theory, and is found to be by experiment, proportional to the square of the field strength. The effect is dependent upon temperature, decreasing rapidly with a rise in temperature. The Cotton-Mouton effect is the magnetic analogue of the electro-optic Kerr effect to be discussed in Sec. 32.10 and is not related to the Zeeman effect.

32.6 KERR MAGNETO-OPTIC EFFECT

In 1888 Kerr* made the discovery that when plane-polarized light is reflected at normal incidence from the polished pole of an electromagnet, it becomes elliptically polarized to a slight degree, with the major axis of the ellipse rotated with respect to the incident vibrations. At other angles of incidence the effect is observable if one avoids the ordinary effect of elliptical polarization obtained by reflection of plane-polarized light from metals at $\phi \neq 0$ by having the electric vector of the incident light either parallel or perpendicular to the plane of incidence. Under these conditions, and without the field, the reflected beam can be extinguished by a nicol prism. Upon turning on the magnetic field the light instantly appears and cannot be extinguished by a rotation of the nicol. The introduction of a quarter-wave plate suitably oriented will now enable the light to be again extinguished, showing the reflected light to be elliptically polarized. The magnetic field has thus given rise to a vibration component called the Kerr component perpendicular to the incident light vibration. This is the Kerr magneto-optic effect and should be distinguished from the Kerr electro-optic effect considered in Sec. 32.10.

32.7 STARK EFFECT

In the few years following Zeeman's discovery of the splitting up of spectral lines in a magnetic field, many attempts were made to observe an analogous effect due to an external electric field. In 1913 Stark observed that when the hydrogen spectrum is excited in a strong electric field of 100 kV/cm, each line split into a symmetrical pattern. A photograph of the effect in the first line of the Balmer series of hydrogen is shown in Fig. 32J. When viewed perpendicular to the electric field, some of the components of each line pattern are observed to be plane-polarized with the electric vector parallel to the field ($p$ components) and the others plane-polarized with the electric vector normal to the field ($s$ components). This is the transverse Stark effect. When viewed parallel to the field, only the $s$ components appear, but as ordinary unpolarized light. This is the longitudinal Stark effect.

The theory of the Stark effect has been developed only in terms of the quantum theory and will not be given here.†

* John Kerr (1824-1907), pronounced "car," Scottish physicist, inspired to investigate electricity and magnetism by his association with William Thomson (Lord Kelvin).
The method used for producing strong electric fields of 100kV/cm or more, in which the light source will operate, is based upon the characteristics of the ordinary discharge of electric currents through gases at low pressures. In a discharge of the type shown in Fig. 21D, the major part of the potential drop from one electrode to the other occurs across a relatively dark region near the cathode. This region of a specially designed discharge tube, when focused on the slit of a spectrograph, may be made to give photographs of the type shown in Fig. 32K. Since the Stark effect is proportional to the field $F$, the pattern of $\lambda 3819$, for example, may be taken to represent the field strength which is small at the top and increases toward the bottom, nearer the cathode.

The widest Stark patterns are observed in the hydrogen and helium spectra. In the case of all other spectra one seldom observes anything but a slight shift of the line, usually toward longer wavelengths. This effect is called the quadratic Stark effect, to distinguish it from the linear effect observed in hydrogen and helium. In the former case the shifts are proportional to the square of the electric field strength, while in the latter the splittings depend on the first power of this field. Characteristic of the Stark effect, as shown in Fig. 32K for the helium spectrum, is the appearance of new spectrum lines (marked with crosses) where the field strength is high.

### 32.8 INVERSE STARK EFFECT

The Stark effect with the lines appearing in absorption is called the inverse Stark effect. The phenomenon has been investigated by Grotrian and Ramsauer, using a long tube containing potassium vapor at low pressure and two long parallel metal plates only 1.5 mm apart. With a potential of 14 kV on the plates, the absorption lines $\lambda 4044$, $\lambda 4047$, and $\lambda 3447$ were found to be shifted from the field-free
toward longer wavelength. This shift, although only a few hundredths of an angstrom unit in magnitude, was found to be proportional to the square of the field strength. This is therefore a case of a quadratic Stark effect.

### 32.9 ELECTRIC DOUBLE REFRACTION

Electric double refraction is related to the transverse Stark effect and is analogous to magnetic double refraction, or the Voigt effect, discussed in Sec. 32.4. In 1924 Ladenberg observed the absorption of the sodium resonance lines when produced with and without a strong transverse electric field applied to the vapor. Although the shift of the lines predicted by the quadratic Stark effect was too small to observe even with very high resolving power, double refraction was observed at frequencies close to the absorption lines. This double refraction is attributed to the very small difference in the frequency of the absorption line for light polarized parallel and perpendicular to the electric lines of force. The explanation is therefore analogous to that given for magnetic fields in Sec. 32.4 (see Fig. 321).

### 32.10 KERR ELECTRO-OPTIC EFFECT

In 1875 Kerr discovered that when a plate of glass is subjected to a strong electric field, it becomes doubly refracting. That this effect is not due to the strains that such a field sets up in the glass is shown by the fact that the phenomenon also appears in many liquids and may even be observed in gases. When a liquid is placed in an
electric field, it behaves optically like a uniaxial crystal with the optic axis parallel to the field direction, and when viewed from the perpendicular direction, it gives rise to all the phenomena of interference considered in Chap. 27.

It is convenient experimentally to observe the effect by passing light between two parallel oppositely charged plates inserted in a glass cell containing the liquid. Such a device, known as a Kerr cell, is shown at the center in Fig. 32L. Such a cell inserted between crossed polarizer and analyzer constitutes a very useful optical device known as the electro-optic shutter.* One of these uses was described in Sec. 19.5. When the electric field is off, no light is transmitted by the analyzer. When the electric field is on, the liquid becomes doubly refracting and the light is restored. With the cell oriented at 45°, the incident plane vibrations from the polarizer are broken up into two equal components parallel and perpendicular to the field, as shown at the bottom in Fig. 32L. These travel with different speeds, and hence a phase difference is introduced and the light emerges as elliptically polarized light. The horizontal component of the vibrations is transmitted by the analyzer.

The change in phase of the two vibrations in a Kerr cell is found to be proportional to the path length, i.e., the length of the electrodes \( l \) and to the square of the field strength \( F \). The magnitude of the effect is determined by the Kerr constant \( K \), defined by the relation

\[
\Delta = K \frac{lE^2\lambda}{d^2} \tag{32i}
\]

Since the phase difference \( \delta \) between the two components is given by \( 2\pi/\lambda \) times the path difference \( \Delta \), we have

\[
\delta = K \frac{2\pi lE^2}{d^2} \tag{32j}
\]

where \( \delta \) is in radians, \( l \) and \( d \) are in meters, \( E \) is in volts, and \( K \) is in meters per volts squared, and \( \lambda \) is the wavelength in the medium.

One of the substances most suitable for use in a Kerr cell is nitrobenzene, because of its relatively large Kerr constant. This is shown by the values given for a few liquids in Table 32B.

It should be pointed out that electric double refraction for gases discussed in the last section and the Kerr electro-optic effect are not the same phenomenon. In a gas the effect is due to changes inside the atom (Stark effect). In the Kerr effect it is usually due to natural or induced anistropy of the molecule and a lining up of such molecules in the field. This alignment causes the medium as a whole to be optically anistropic. As in the Cotton-Mouton effect (Sec. 32.5), the Kerr effect is dependent on temperature. In fact, the Kerr electro-optic effect is the exact electric analogue of that magnetic effect.

### 32.11 Pockels Electro-Optic Effect

A variety of uniaxial crystals have been found in which the induced birefringence varies linearly with the applied electric field. This effect was named after F. Pockels,* who studied the effect in 1893. Recent research has developed a variety of electro-optic crystals, such as ammonia dihydrogen phosphate \((\text{NH}_4\text{H}_2\text{PO}_4)\) (ADP) and potassium dihydrogen phosphate \((\text{KH}_2\text{PO}_4)\) (KDP), which produce sizable Pockels birefringence at relatively low voltages (see Fig. 32M).

A Pockels cell, which can be used as a fast light modulator or shutter, usually involves a crystal mounted with its optic axis and applied field parallel to the beam direction (see Fig. 32N). By placing the cell between crossed polarizers, the transmission can be modulated at frequencies well above \(10^{10}\) Hz, and as a shutter with a response time shorter than 1 ns. Since the beam traverses the electrodes, these are frequently made of transparent metallic oxides, such as CdO, SnO, or InO, or thin metallic rings or grids.

Pockels cells, like Kerr cells, are used for a wide range of electro-optic systems, including their use as a Q switch to produce ultrashort laser pulses (see Sec. 30.6). These systems have been proposed as wideband laser-beam communication systems to be used, in addition to terrestrial applications, in interplanetary space.

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### Table 32B Values of the Kerr Constant for \(\lambda = 5893\) Å

<table>
<thead>
<tr>
<th>Substance</th>
<th>(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>(0.67 \times 10^{-14})</td>
</tr>
<tr>
<td>Carbon disulfide</td>
<td>(3.56 \times 10^{-14})</td>
</tr>
<tr>
<td>Water</td>
<td>(5.10 \times 10^{-14})</td>
</tr>
<tr>
<td>Nitrotoluene</td>
<td>(1.37 \times 10^{-14})</td>
</tr>
<tr>
<td>Nitrobenzene</td>
<td>(2.44 \times 10^{-14})</td>
</tr>
</tbody>
</table>
FIGURE 32M
A laboratory-grown crystal of ammonia dihydrogen phosphate (NH₄H₂PO₄), or ADP, for use in Pockels cells.

FIGURE 32N
Components of a Pockels cell for high-frequency modulation of a light beam.
PROBLEMS

32.1 Determine the Zeeman splitting $\Delta \sigma$ of a single line in the zinc spectrum, where the wavelength is 4700 Å. Express this splitting in angstroms and assume the field strength to be 2.520 T.  
Ans. $\Delta \lambda = 0.260 \ \text{Å}$

32.2 The photograph of the normal Zeeman effect shown in Fig. 31D(a) was enlarged 20 times from the original negative. The plate factor of the spectrograph used was 2.30 Å/mm at the wavelength of the line 4700 Å. What was the value of the magnetic induction?

32.3 For the first line of the Paschen series of hydrogen, the wavelength is 18,746 Å. Calculate the Zeeman shifts for a normal Zeeman triplet if the magnetic field is 1.650 T.

32.4 A diffraction grating has 50,000 lines ruled on its surface. What strength of magnetic field would have to be applied to a light source for the grating to be able to resolve a normal Zeeman triplet (a) in the violet at 4500 Å and (b) in the red at 6500 Å? Assume the first-order spectrum is used.  
Ans. (a) 0.952 T (b) 0.659 T

32.5 In the Faraday effect a magnetic field of 0.64 T is applied to a piece of light flint glass 10.50 cm long. Find the angle of rotation in degrees.

32.6 The Faraday effect is performed with a liquid in a glass tube 20.0 cm long. If the applied magnetic induction is 0.820 T and the measured rotation of plane polarized light is 65.46°, what is the value of the Verdet constant?

32.7 The Faraday effect is performed with a piece of phosphate crown glass 5.0 cm thick. This glass is placed between Polaroids with their principal sections at 45° with each other. (a) What magnetic field strength applied to the glass will rotate the plane of polarization 45° in order that the light transmitted gets through with maximum intensity? (b) If ordinary light is sent through the system in the reverse direction, what will be the intensity of the transmitted light? (c) Is this a one-way optical system?  
(d) Make a diagram.

32.8 Very pure nitrobenzene is used in a Kerr cell, with a power source of 20 kV applied to its plates. If the plates of the cell are 2.5 cm long and 0.75 cm apart, find (a) the phase difference between the components emerging from the cell. If unpolarized light is incident on the polarizer, what is (b) the amplitude of the plane-polarized light incident on the cell, (c) the amplitude of the light emerging from the analyzer, and (d) the intensity of the emerging light?  
Ans. (a) 156.44°, (b) 0.7071A_o, (c) 0.6920A_o, (d) 0.4790I_o

32.9 What voltage applied to a Kerr cell will produce polarized light emerging from the cell? The electrodes are 3.0 cm long and 5.0 mm apart, and the cell is filled with nitrotoluene.

32.10 A Kerr cell using very pure nitrobenzene has plates that are 2.80 cm long, separated by a distance that is 0.60 cm. (a) What voltage should be applied to the plates to produce a maximum in the transmitted intensity? (b) At this field strength, what fraction of the incident unpolarized light will get through the system? Neglect losses by reflection and absorption.